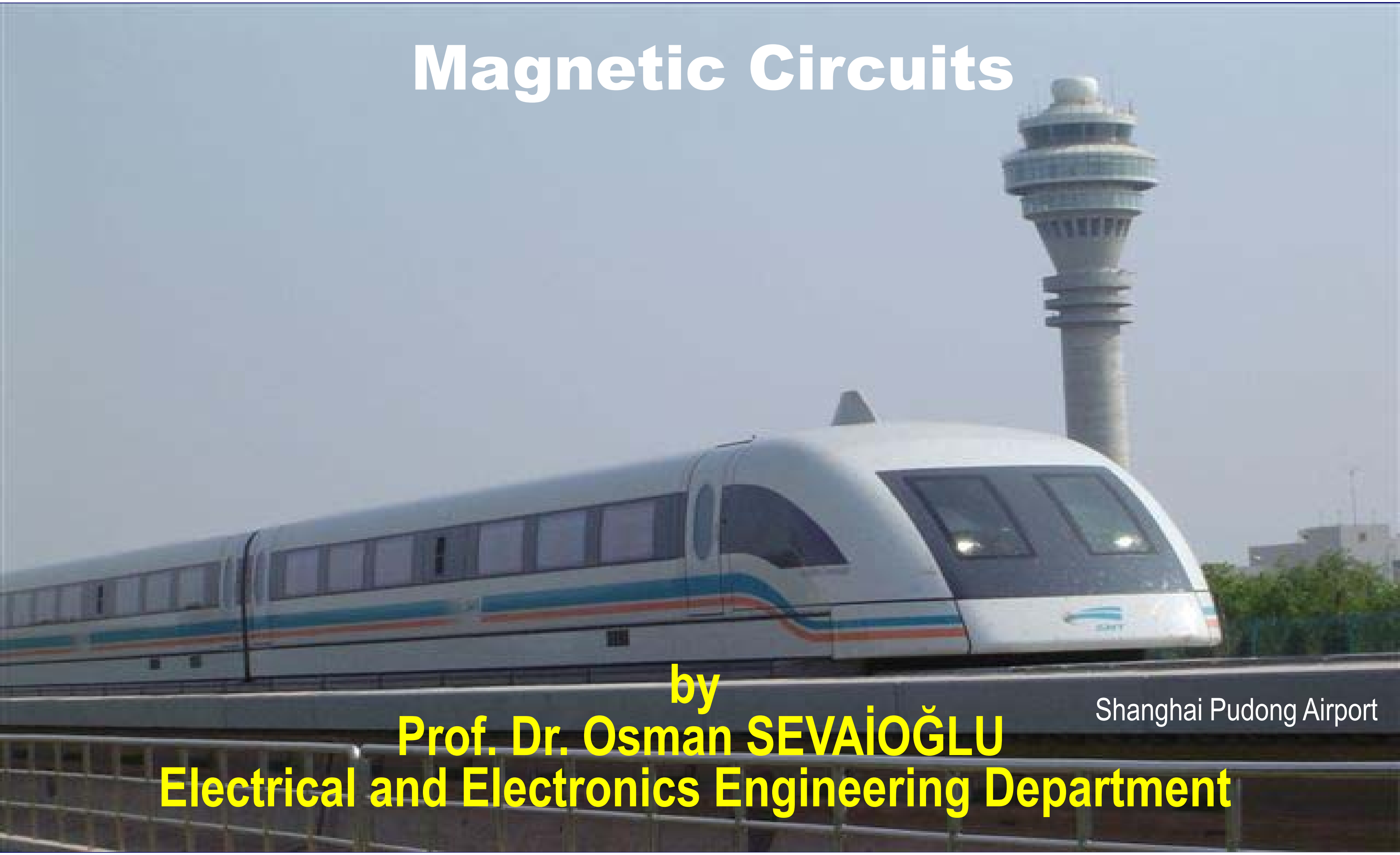


Magnetic Circuits

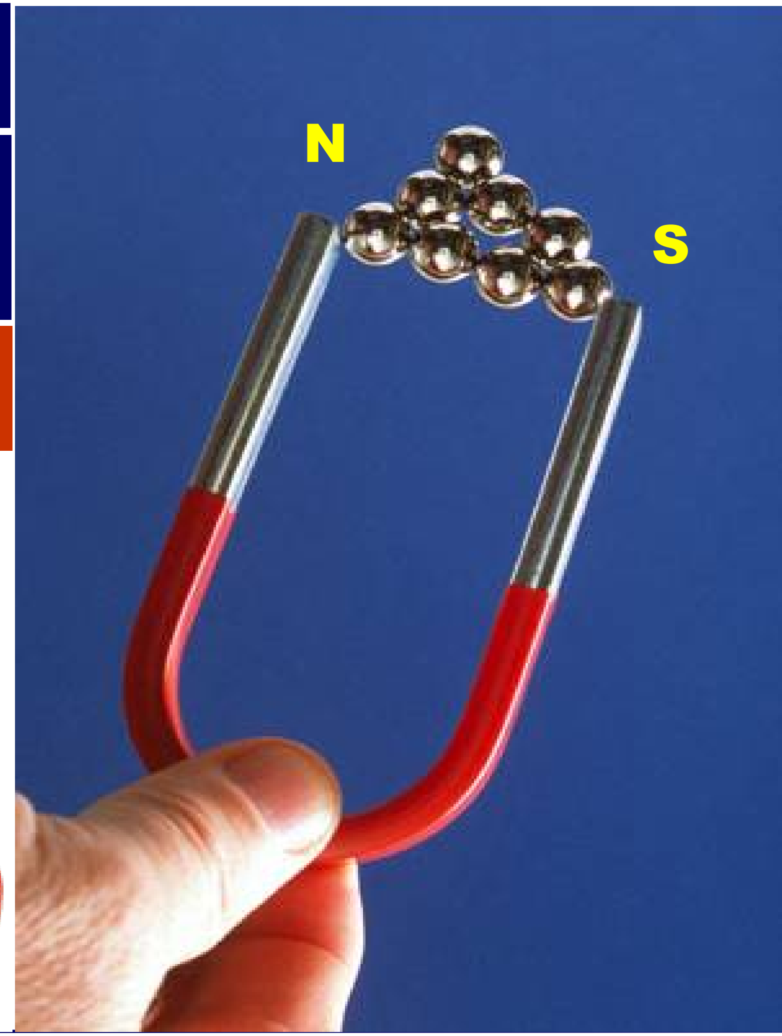
by
Prof. Dr. Osman SEVAİOĞLU
Electrical and Electronics Engineering Department

Principles of Magnetism

Permanent Magnet

A permanent magnet is a piece of metal with characteristics of attracting certain metals

Horseshoe Magnets



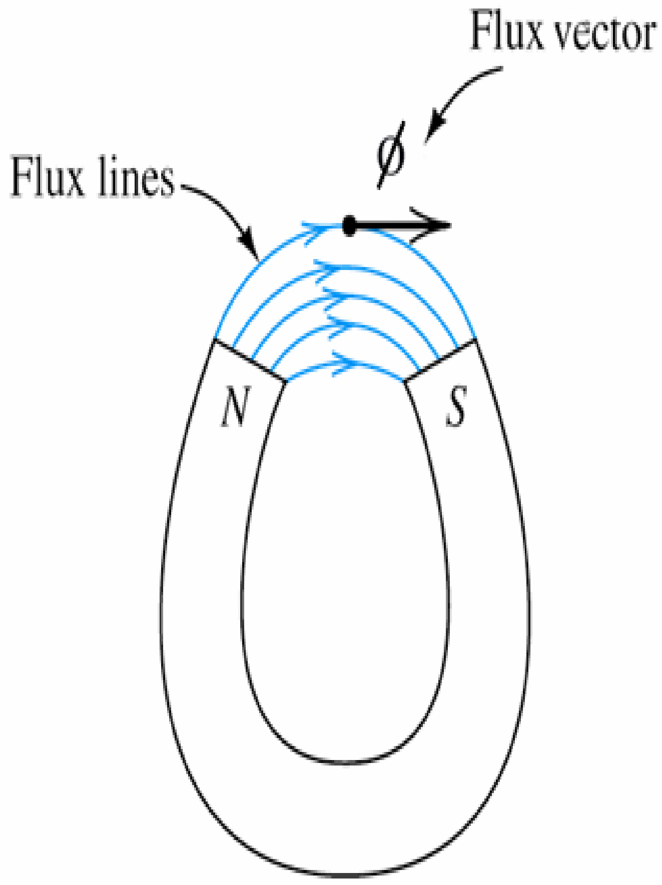
Principles of Magnetism

Permanent Magnet

Magnetic flux lines form closed paths that are close together where the field is strong and farther apart where the field is weak.

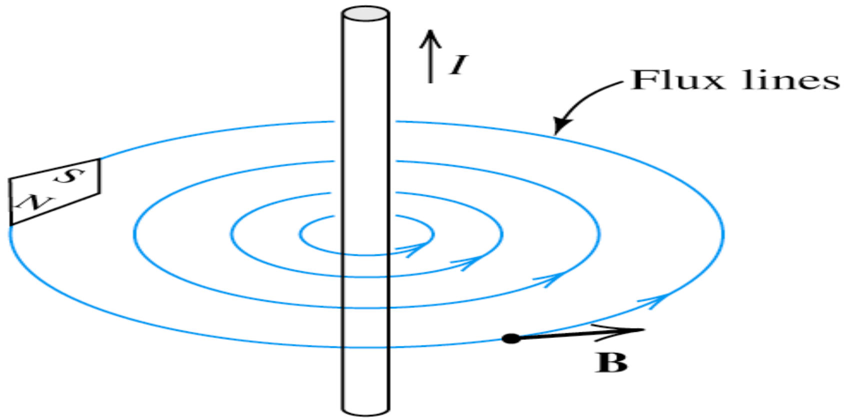
Flux lines leave the north-seeking end of a magnet and enter the south-seeking end.

When placed in a magnetic field, a compass indicates north in the direction of the flux lines.



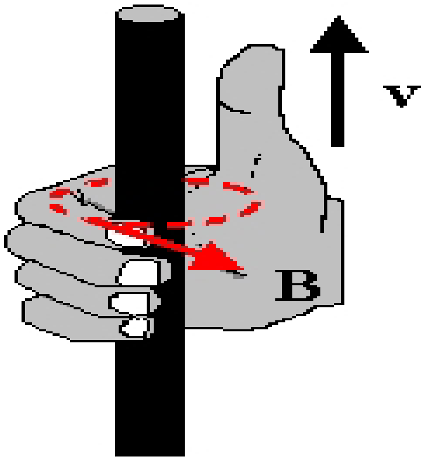
Principles of Electromagnetism

Magnetic Field around a wire	Right Hand Rule
A current in a wire creates a magnetic field around the wire as shown in the following figure	Wrap your right hand fingers aroud the wire while your thumb finger points the direction of current flow



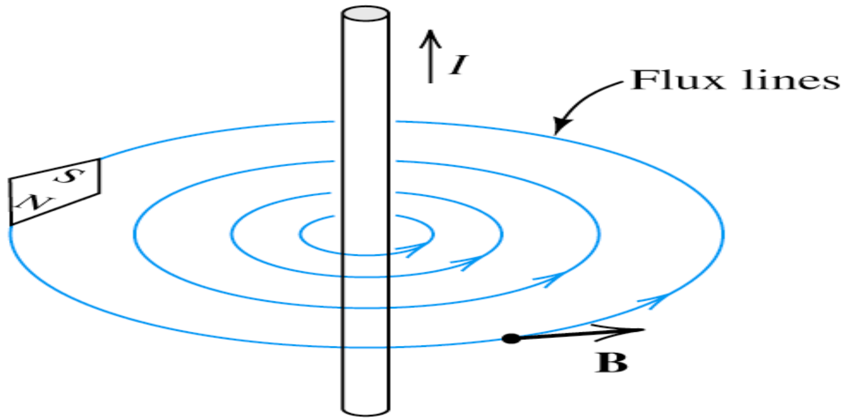
(b) Field around a straight wire carrying current I

Other fingers will point the direction of field lines

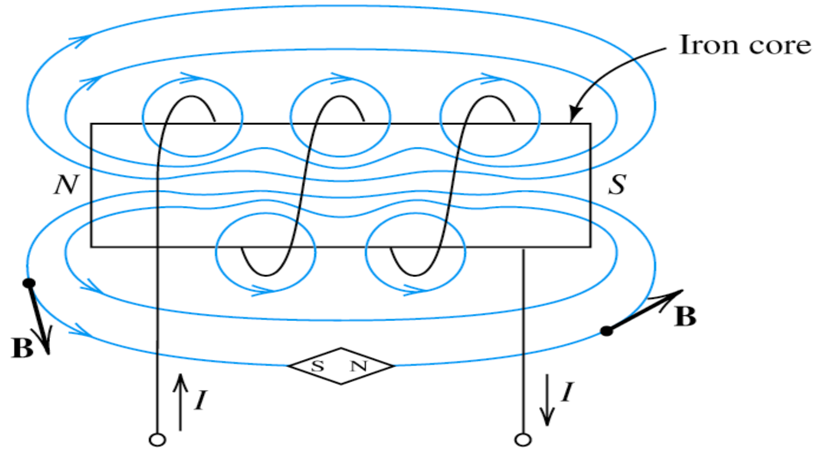


Principles of Electromagnetism

Magnetic Field around a wire	Electromagnet
A current in a wire creates a magnetic field around the wire as shown in the following figure	*If this wire is wound around an iron core, the field lines are superposed as shown in the following figure*



(b) Field around a straight wire carrying current I

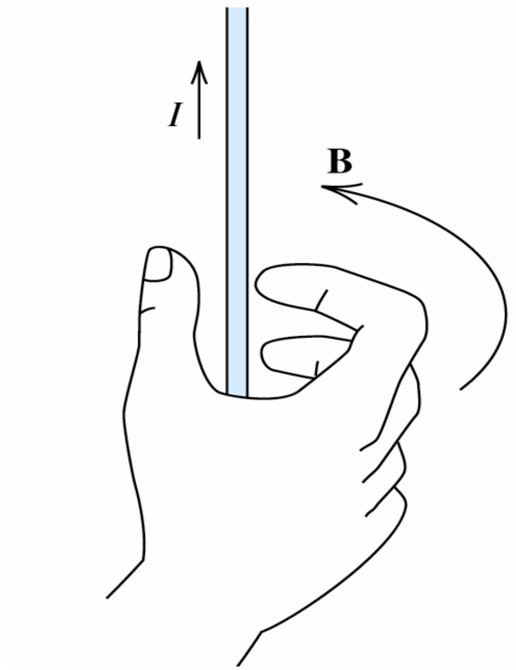


(c) Field for a coil of wire

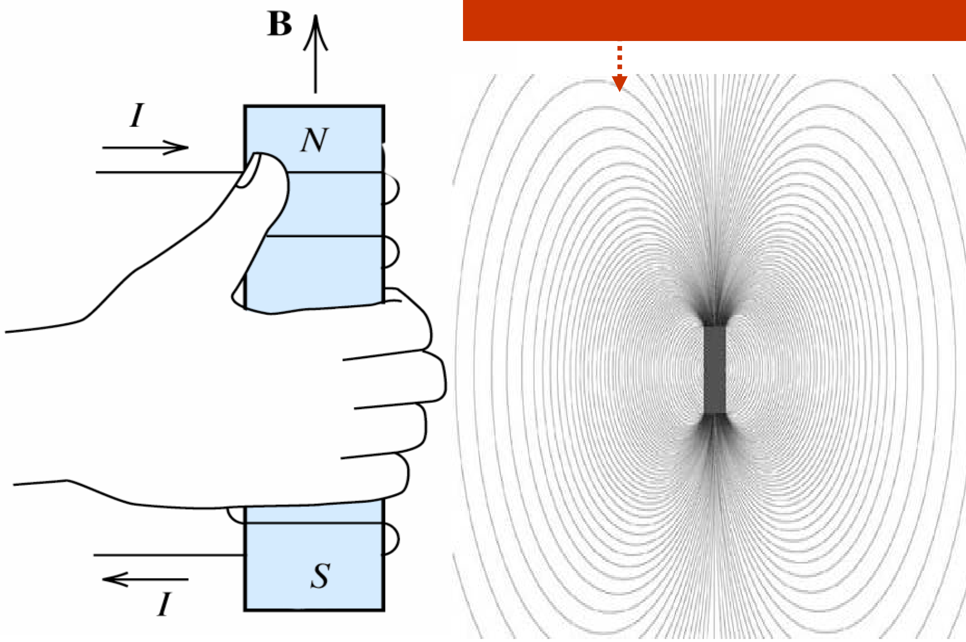
Application of Right Hand Rule to Electromagnets

Wire is grasped with the thumb finger pointing the direction of current flow, the fingers encircling the wire point the direction of the magnetic field

The coil is grasped with the fingers pointing the direction of current flow, the thumb finger points the direction of the magnetic field in the coil

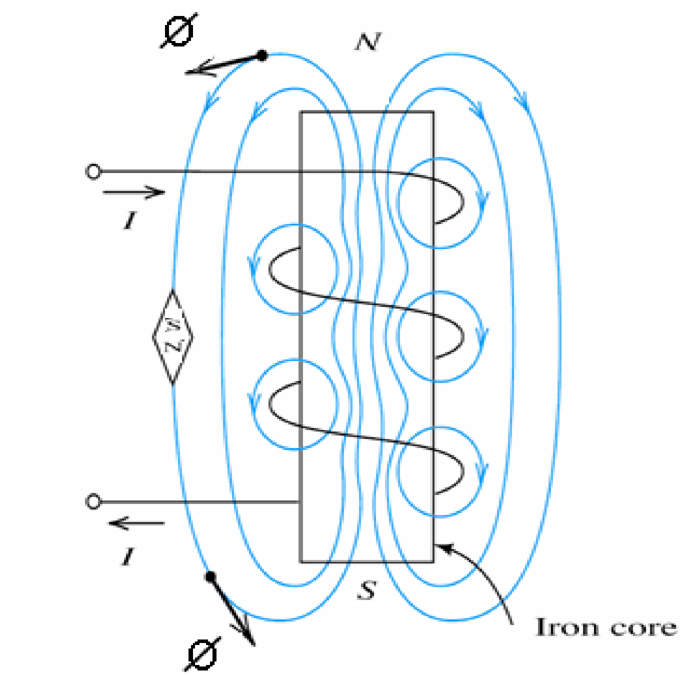


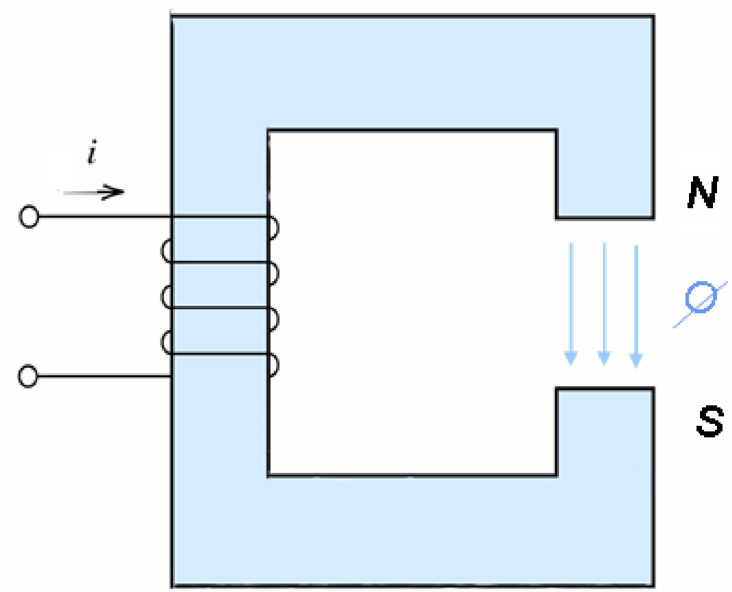
Field lines



Electromagnet

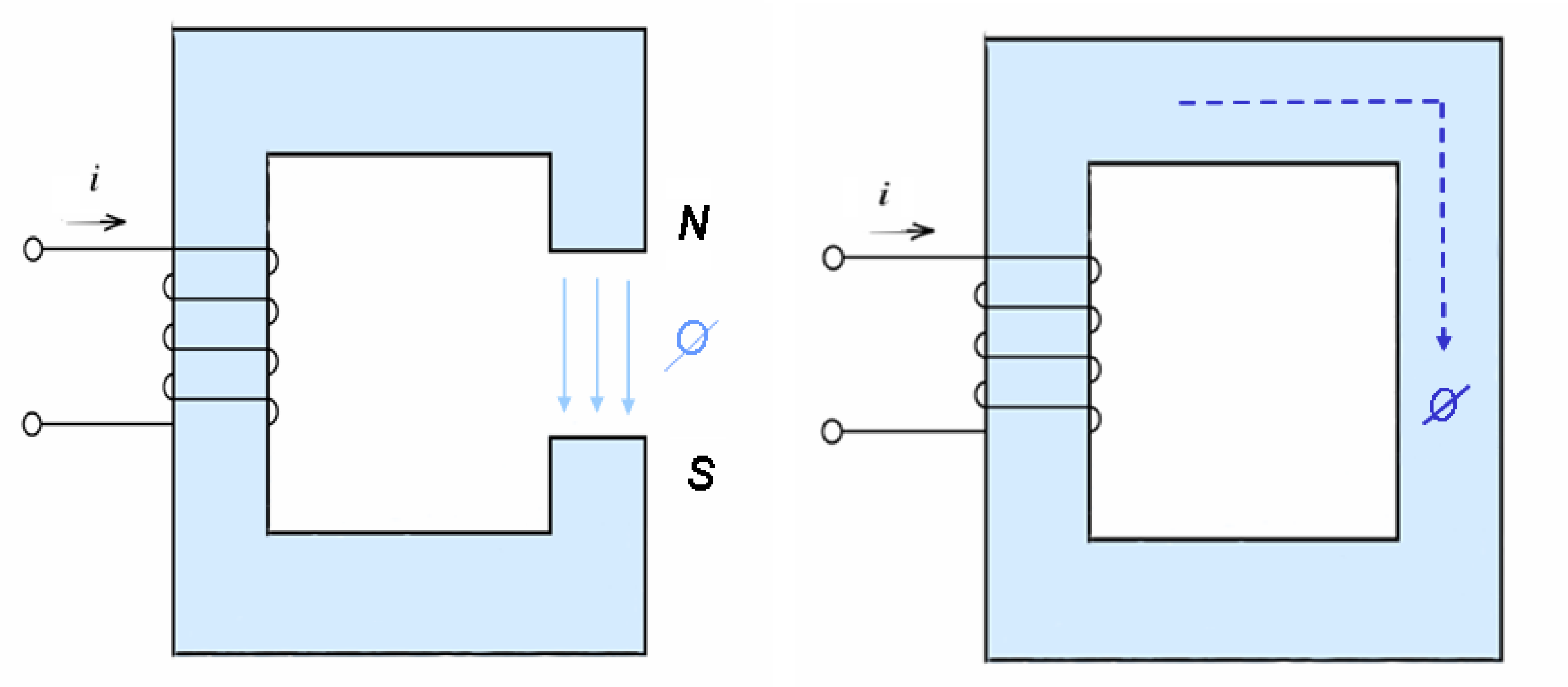
Electromagnet	Horseshoe Electromagnet
Consider again the coil wound around the iron core as shown	*If the iron core is shaped as an "U" shape , we obtain a "horseshoe" electromagnet*





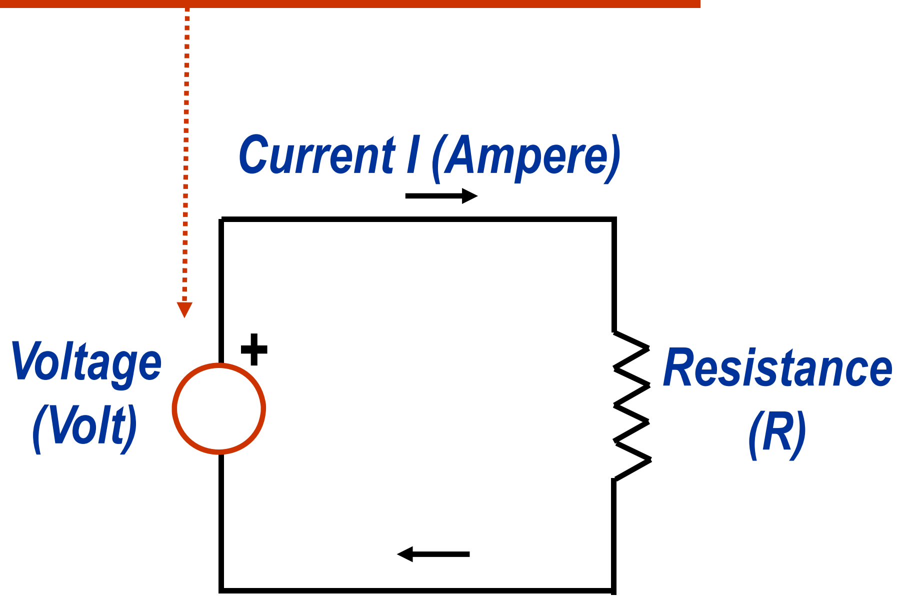
Electromagnet

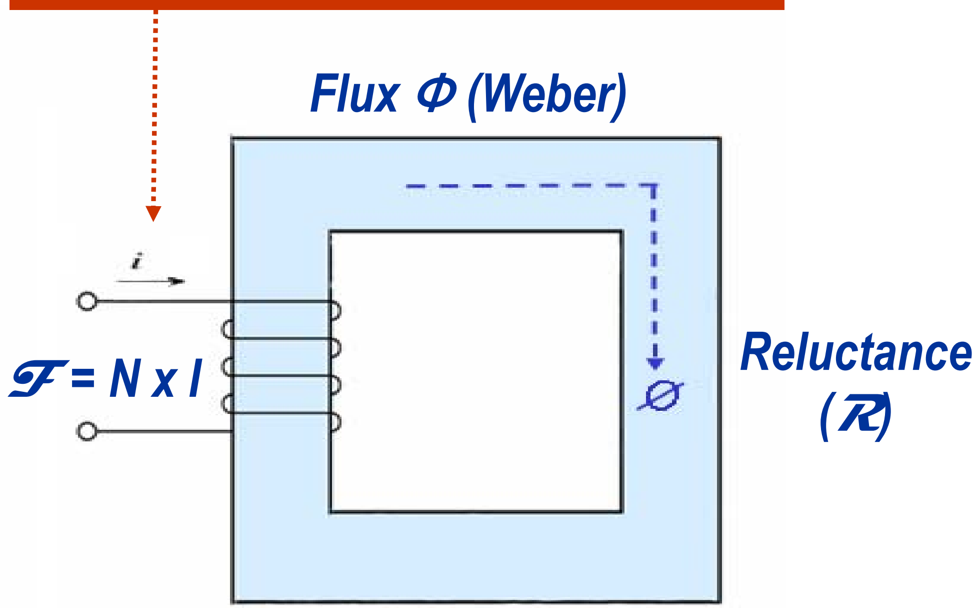
Iron Core may be closed to form a magnetic "Loop"



Magnetic Ohm's Law

Electrical Ohm's Law	Magnetic Ohm's Law
Electro Motive Force (emf)	*Magneto Motive Force (mmf)*





$V = R \times I$	$\mathcal{F} = \mathcal{R} \times \Phi$
(Volt) (Ohm) (Ampere)	(Ampere x Turn) ( ) (Weber)

Magneto Motive Force (mmf)

Definition

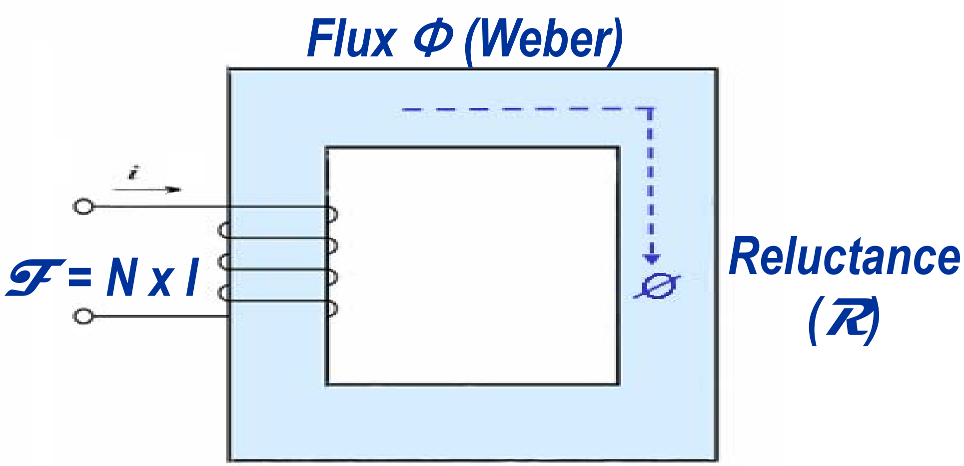
Please note that flux is proportional to both;

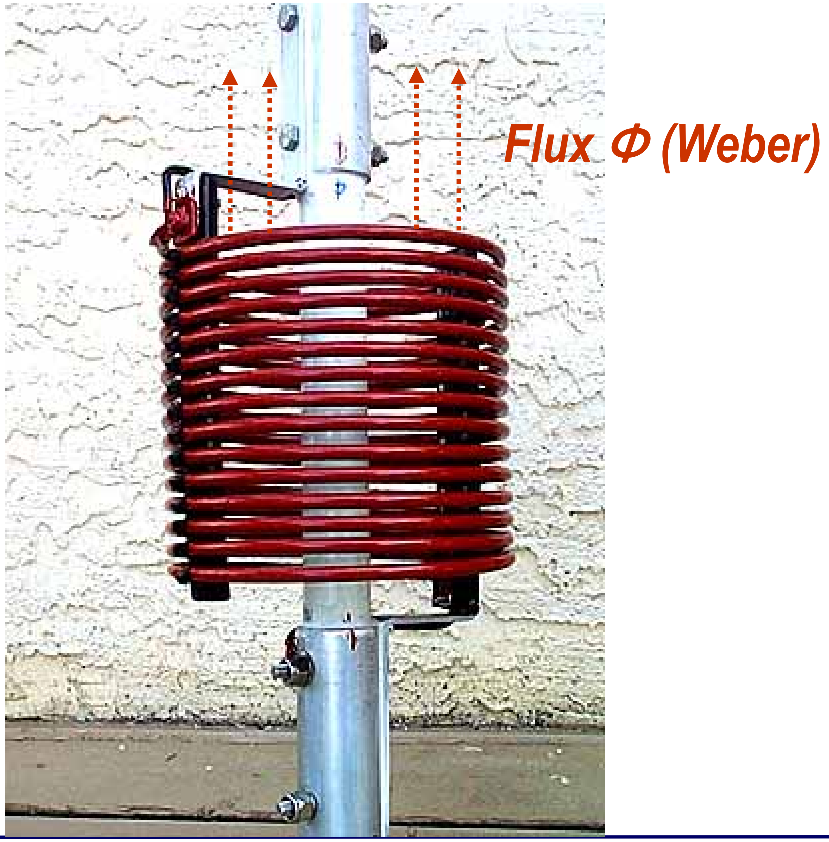
- *Current I,*
- *Number of Turns, N,*

i.e. flux depends on the product: N x I, called Magneto Motive Force (mmf)

$$\mathcal{F} = N \times I$$

(Ampere x Turn) (Turn) (Amper)





Electrical Resistance

Electrical Resistance

Resistance of a cable is proportional to the length and inversely proportional to the cross sectional area of the cable

$$R = \rho \, \ell / A$$

where, R is the resistance of conductor,

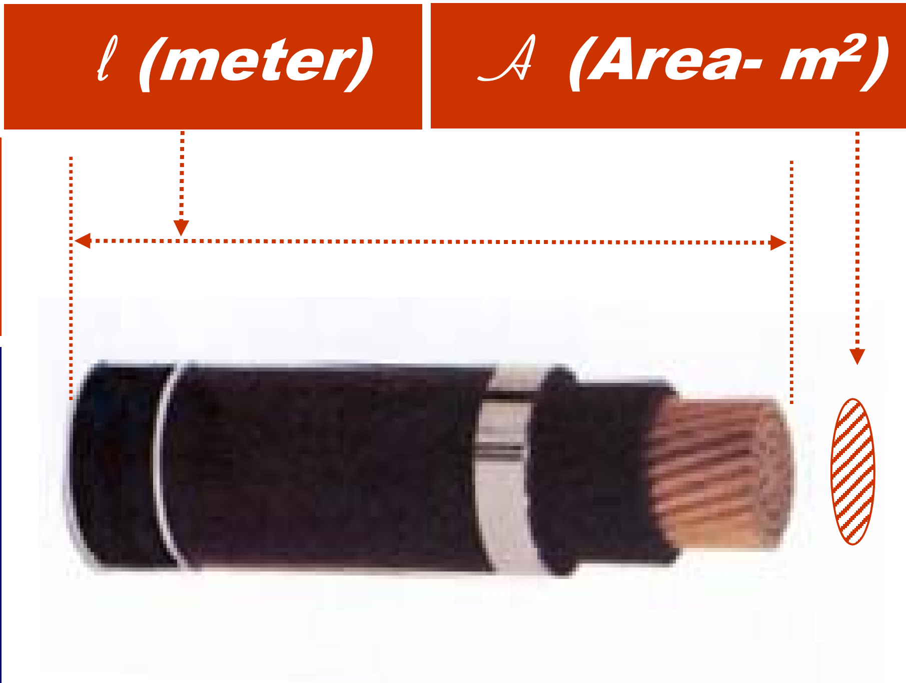
ρ is the resistivity coefficient,

ρ = 1 / 56 Ohm-mm^2/m (Copper)

1 / 32 Ohm-mm^2/m (Alumin.)

l (m) is the length of the conductor

A (mm^2) is the cross sectional area of conductor



Magnetic Resistance (Reluctance)

Reluctance

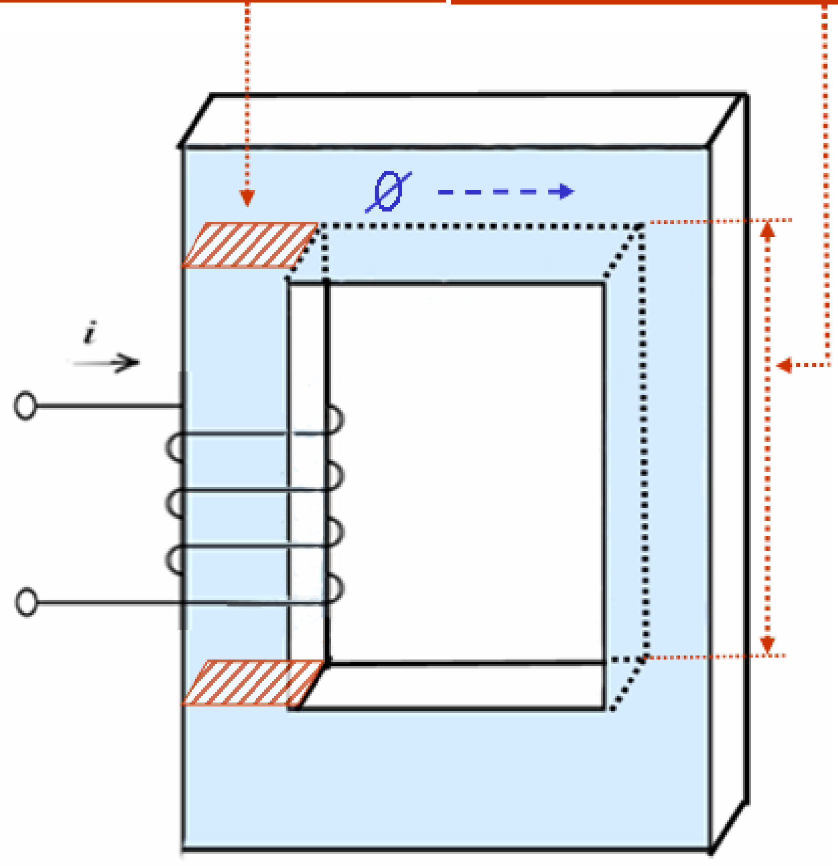
The reluctance of a magnetic material is proportional to the mean length and inversely proportional to the cross sectional area, and the permeability of the magnetic material

$\mathcal{R} = (1/\mu)\ \ell / A$

where, R is the resistance of conductor,
μ is the magnetic permeability coefficient,
$\mu_0 = 4\pi 10^{-7}$ (Air)
ℓ (m) is the length of the material,
A (mm^2) is the cross sectional area of the material

$\mathcal{A}$ (Area- m^2)

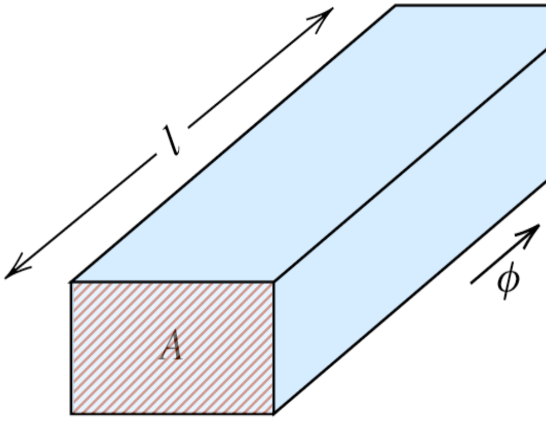
ℓ (meter)



Magnetic Resistance (Reluctance)

Reluctance

The reluctance of a magnetic material is proportional to the mean length and inversely proportional to the product of the cross sectional area and the permeability of the magnetic material

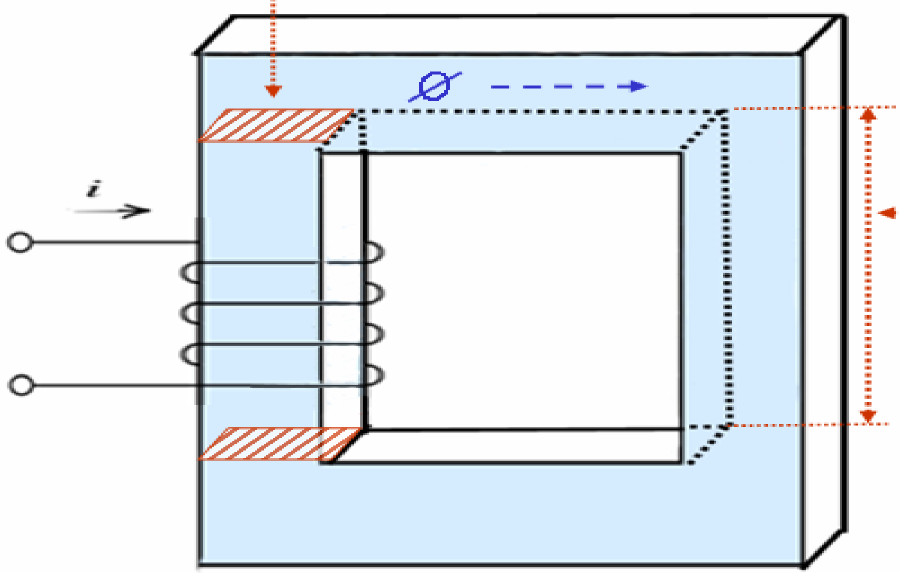


$$\mathcal{R} = \frac{l}{\mu A}$$

Magnetic Circuits

Summary

Resistance		Reluctance	
$\mathcal{A}$ (Area- m^2)	ℓ (meter)	$\mathcal{A}$ (Area- m^2)	ℓ (meter)



$\mathcal{R} = \rho\ \ell / A$	$\mathcal{R} = (1/\mu)\ \ell / A$

Magnetic Circuits

Current Density

Current Density	A (Area- m^2)	I (Amper)

Current density in a cable is the current flowing through per unit area in a plane perpendicular to the direction of current flow

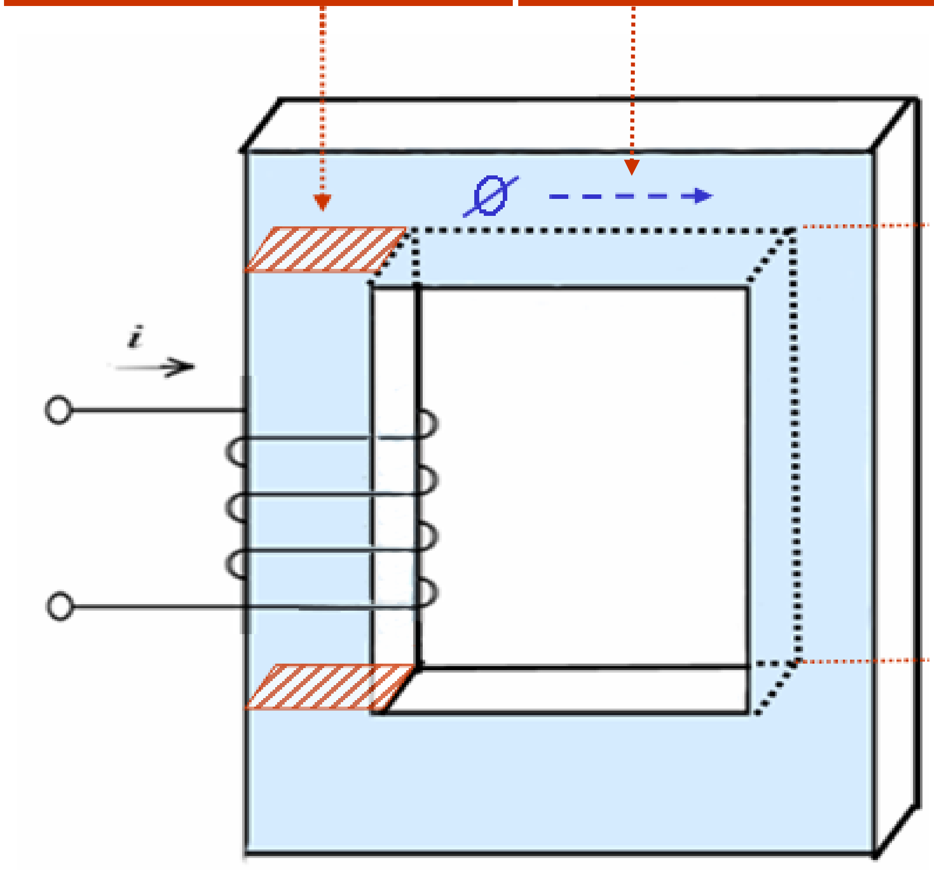
$J = I / A \quad (Amper / m^2)$

Flux Density

Flux Density	A (Area- m^2)	Φ (Weber)

Flux density in a magnetic material is the flux flowing through per unit area in a plane perpendicular to the direction of flux flow

$B = \Phi / A \;\; (\text{Weber} / m^2)$



Magnetic Circuits

Magnetizing Force

Definition

Magnetic Ohm's Law may be rewritten as follows;

$$\mathcal{F} = \mathcal{R} \times \Phi$$

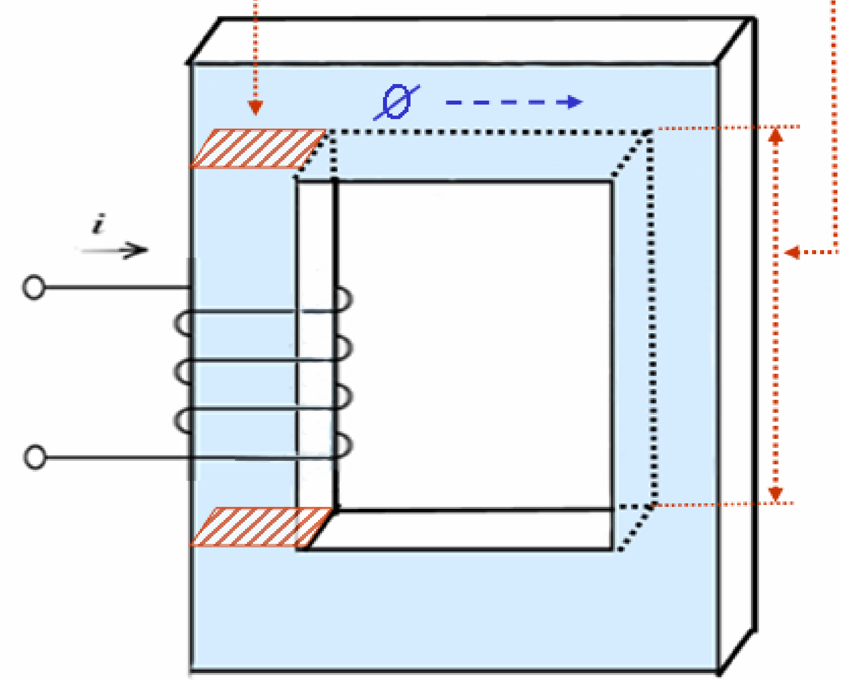
(Ampere x Turn) (Weber)

$$NI = \ell / (\mu A) \times \Phi$$
$$NI = \ell / \mu \times (\Phi / A)$$
$$NI = \ell / \mu \times B$$

or

$$NI / \ell = 1 / \mu \times B$$
$$H = 1 / \mu \times B$$

$\mathcal{A}$ (Area- m^2) | ℓ (meter)



$$B = \mu \times H$$

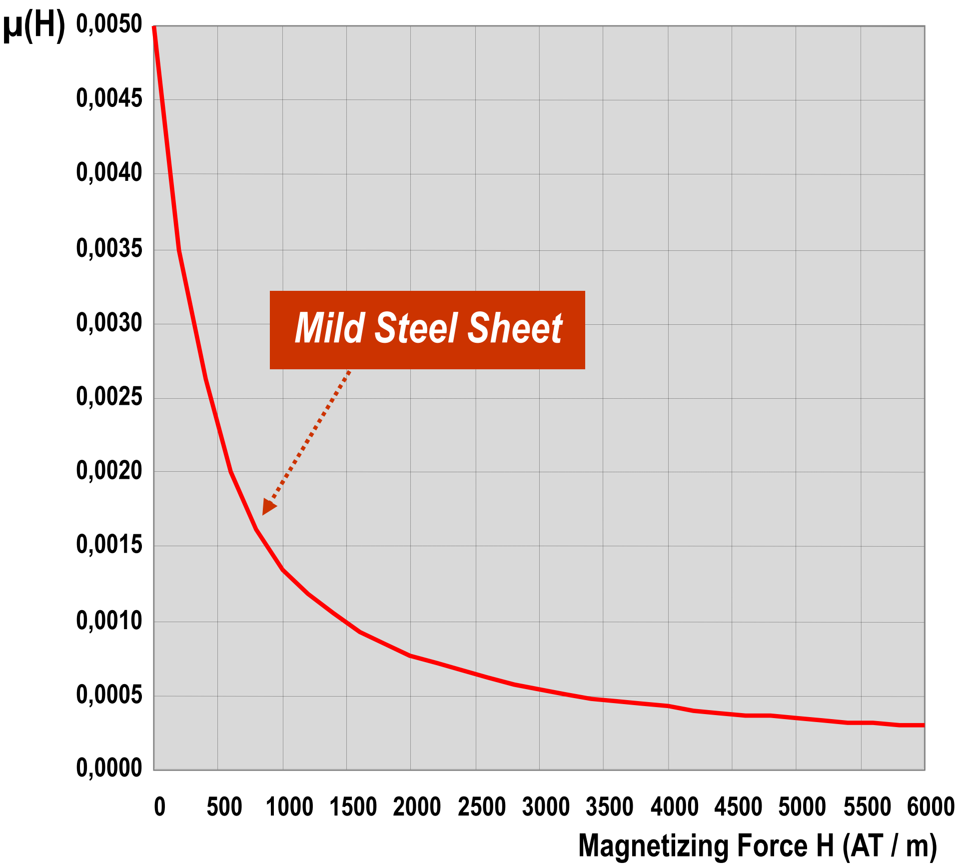
Magnetizing Force

Definition

Please note that the μ coefficient in the following expression is not constant, but function of H.

$$B = \mu \times H$$

μ-H Characteristics



Magnetic Circuits

Magnetizing Force

Definition	$\mathcal{A}$ (Area- m^2)	ℓ (meter)

Magnetizing Force is the mmf per unit length of the magnetic material

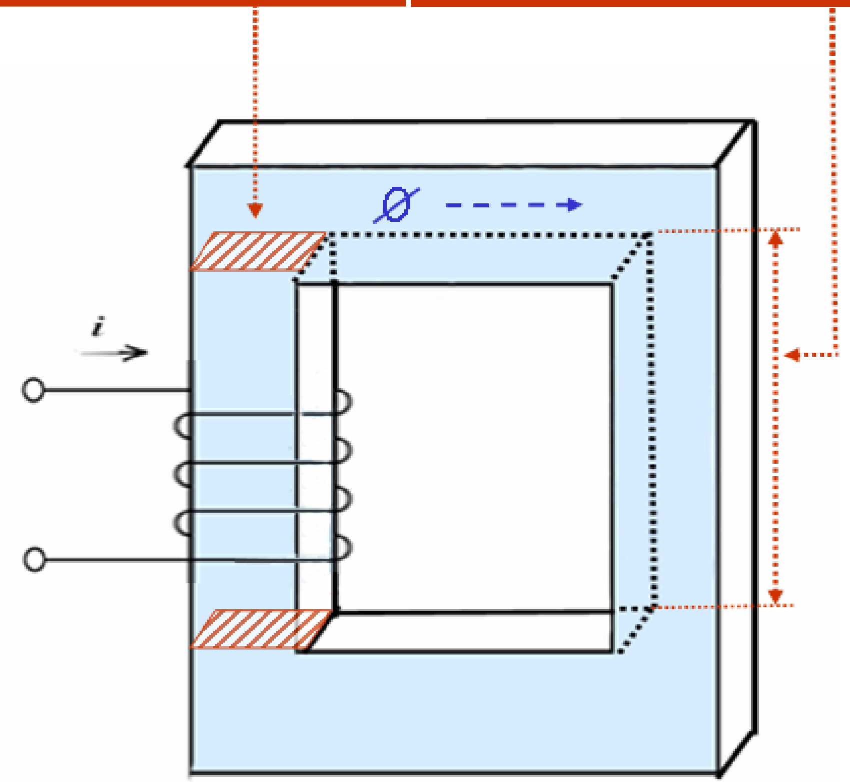
$$NI / \ell = H$$

or

$$H = \mathcal{F} / \ell \qquad (AT / meter)$$

Relation between B and H;

$$B = \mu \times H$$



B-H Characteristics

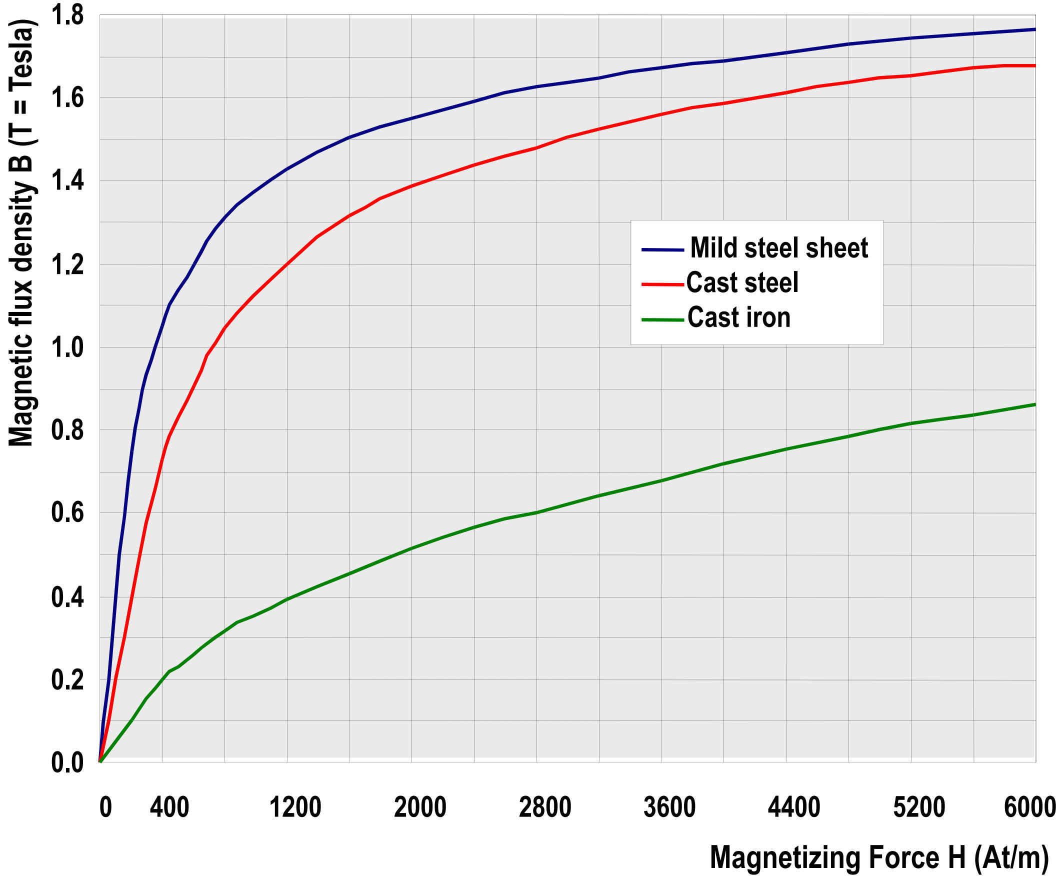
Definition

The importance of B-H characteristics is that, it is independent of the cross sectional area A and length ℓ i.e. it is independent of the shape and volume of the material,

In other words, B-H characteristics exhibits the magnetic property of the material for per unit length and and cross sectional area,

Hence, it is provided by the manufacturer of the magnetic material

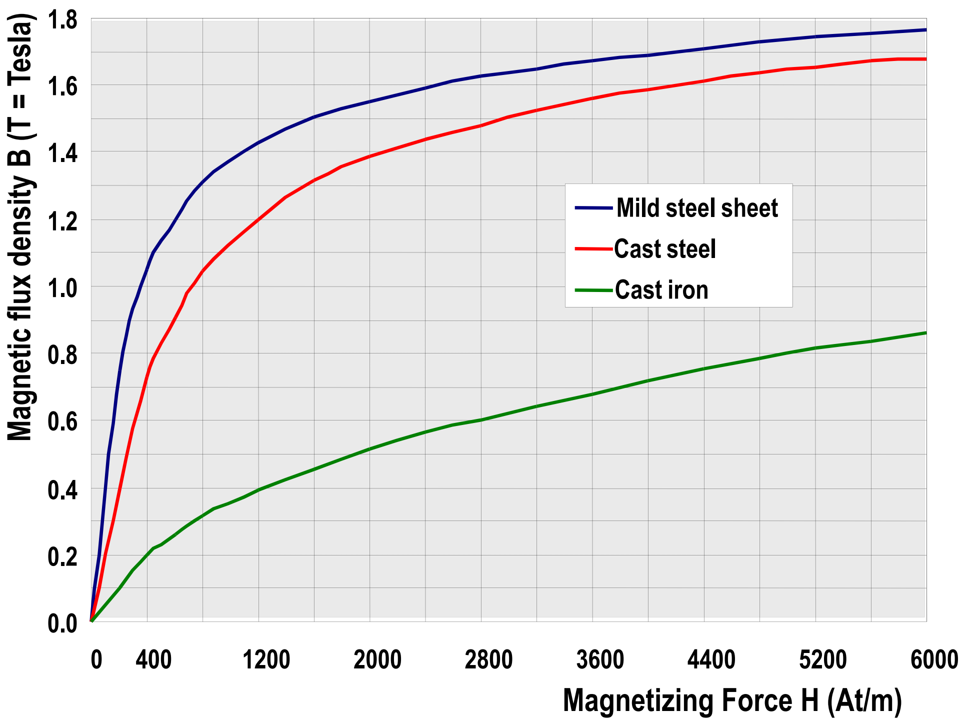
B-H Characteristics



B-H Characteristics

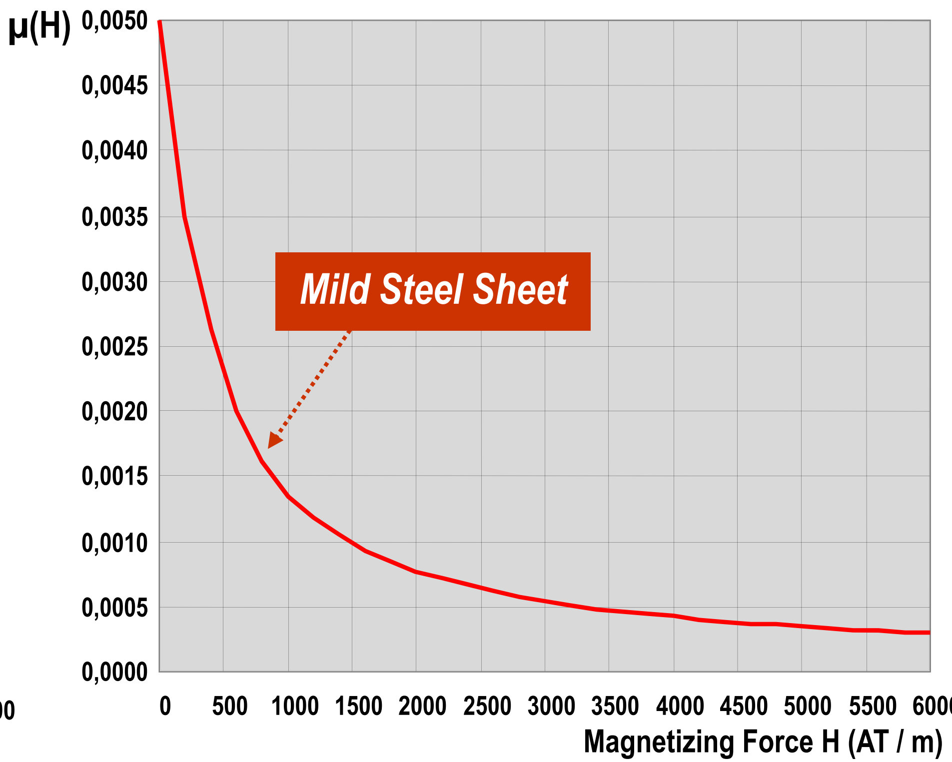
B-H Characteristics

Please note that B-H characteristics saturates at high values of H



μ-H Characteristics

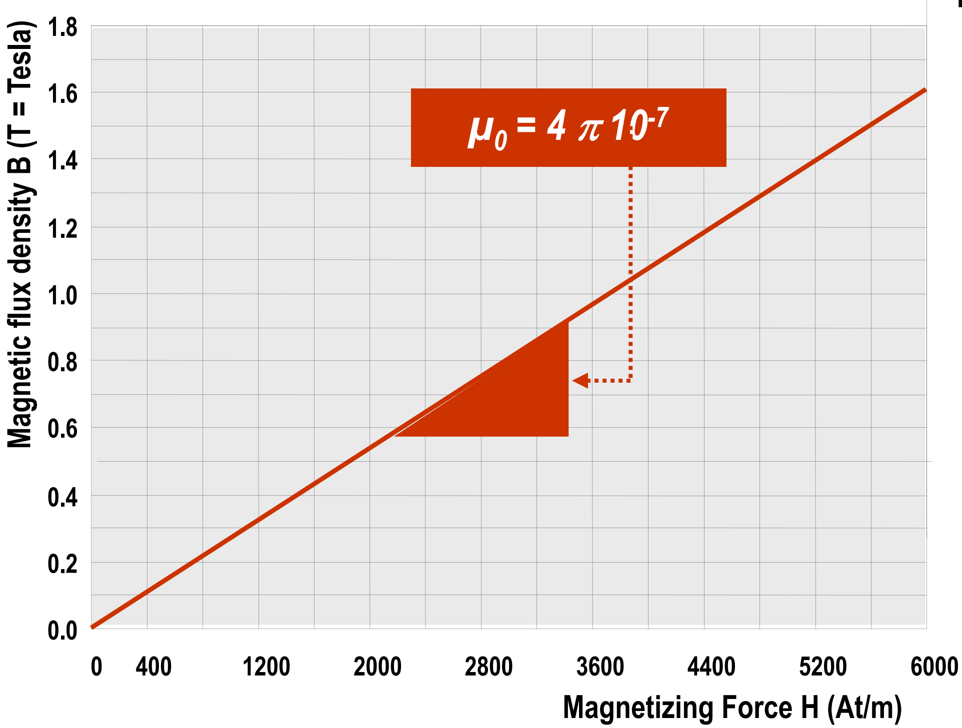
Please note that μ is a function of H



B-H Characteristics of Free Air

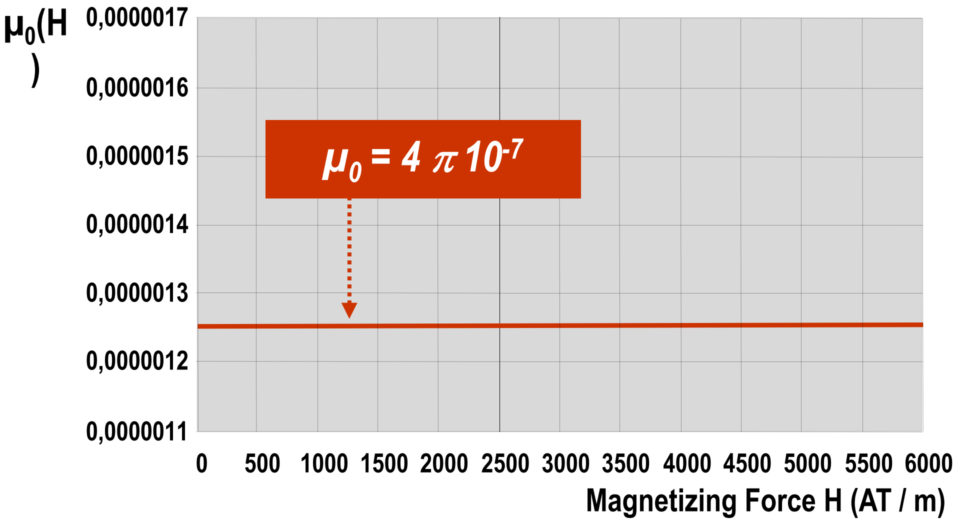
B-H Characteristics

B-H characteristics of free air is linear exhibiting no saturation effect



μ_0-H Characteristics

Please note that μ_0 is constant



The ratio

μ / μ_0

is called **"Relative Permeability"**

Φ-$\mathcal{F}$ Characteristics

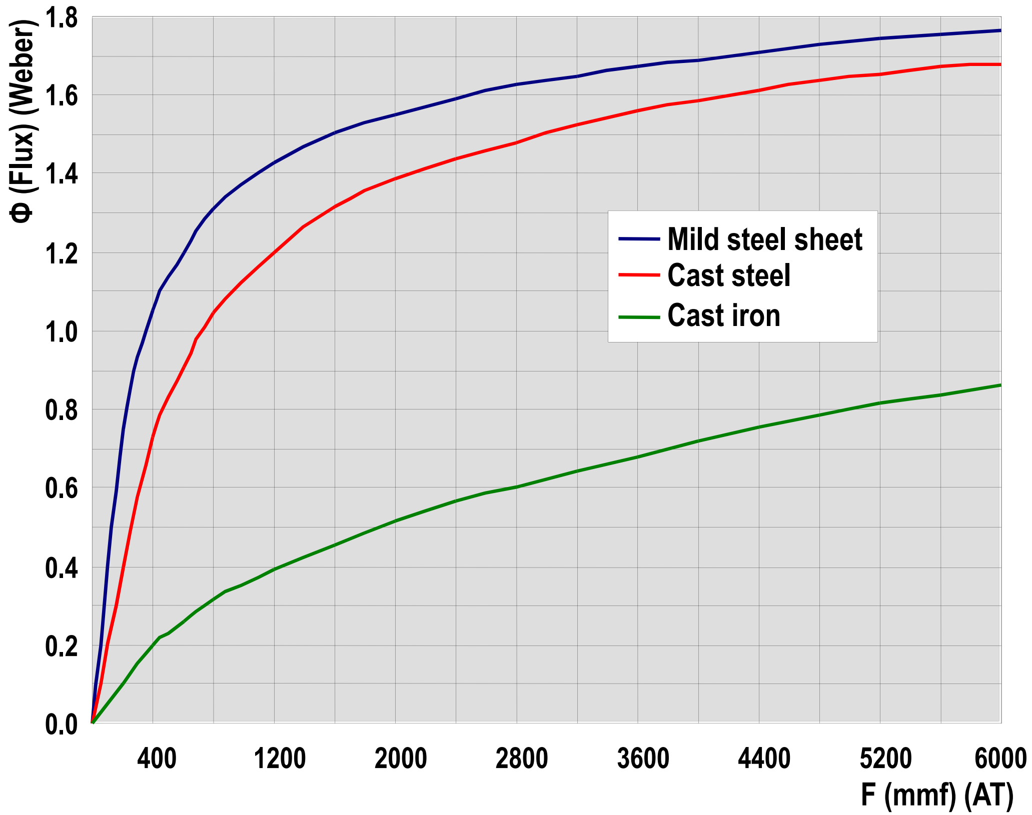
Φ - $\mathcal{F}$ Characteristics

Vertical and horizontal axes in the B-H Characteristics may be multiplied by A and l, respectively yielding the Φ - $\mathcal{F}$ Characteristics

$H \times \ell = \mathcal{F}$ (AT)

$B \times A = \Phi$ (Weber)

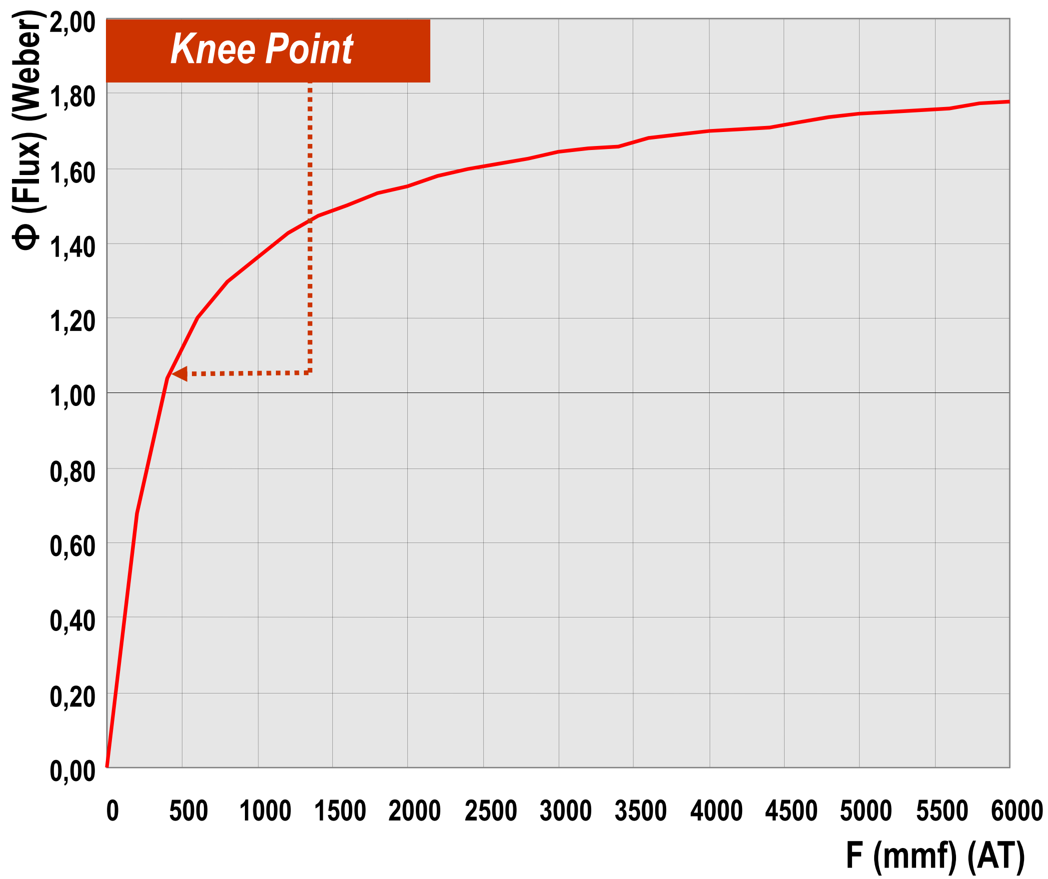
Please note that the shape of the B-H characteristics is unchanged, while only the figures on the axes are changed



Knee Point

Definition	Φ - $\mathcal{F}$ Characteristics

Knee Point on the Φ - $\mathcal{F}$ Characteristics is the point below which the characteristics may be assumed to be linear

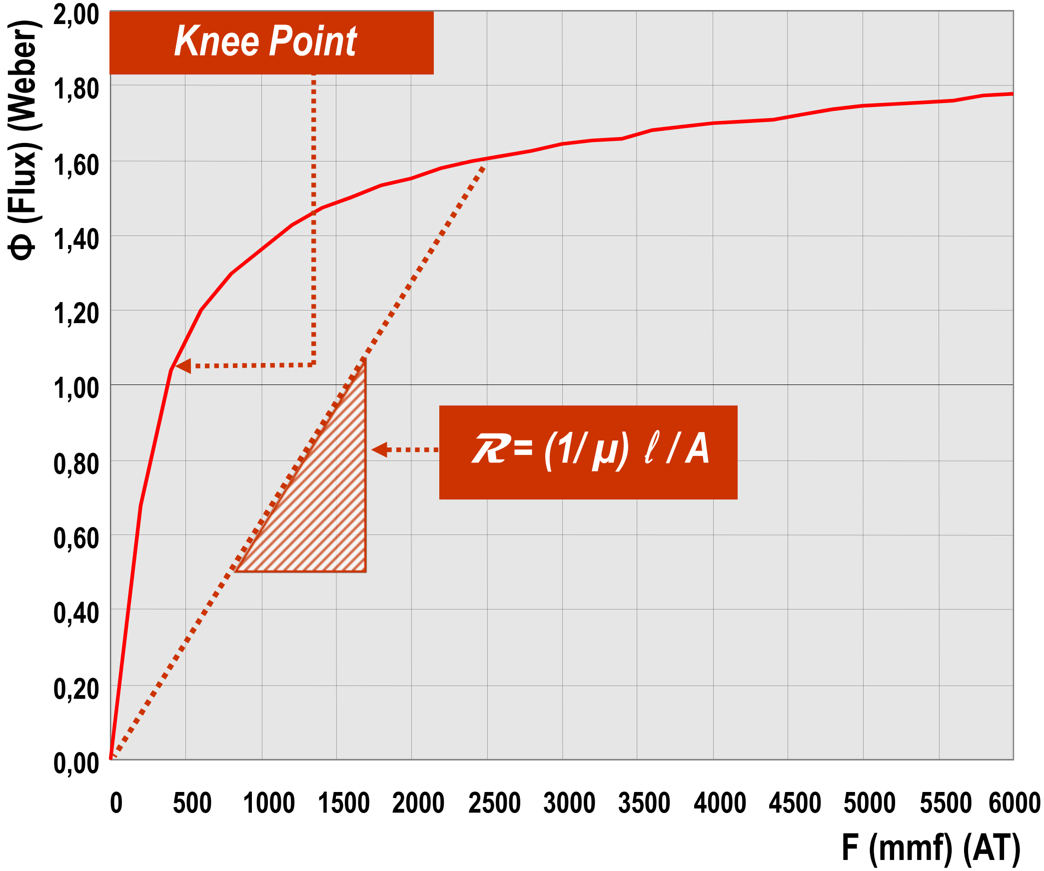


An Alternative Definition of Reluctance

Definition

Reluctance is the inverse of the slope of the chord drawn by joining the origin and a point on the Φ - $\mathcal{F}$ Characteristics

Φ - $\mathcal{F}$ Characteristics

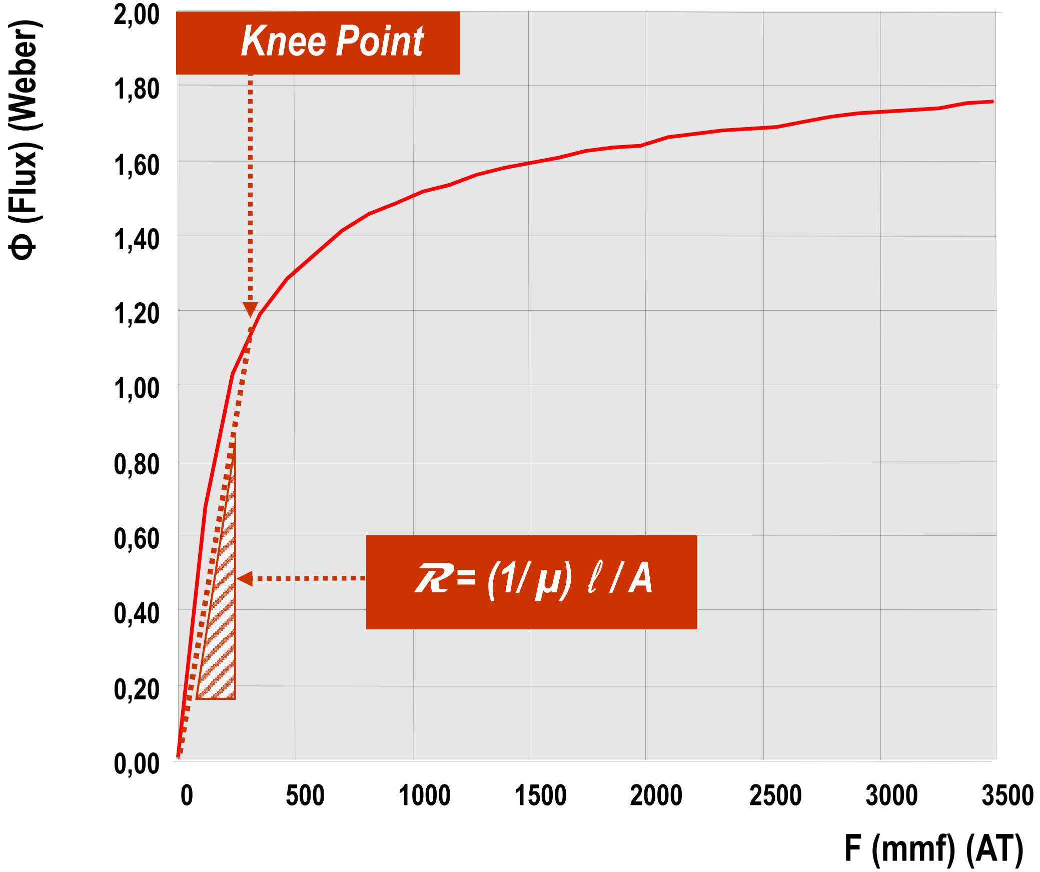


An Alternative Definition of Reluctance

Definition

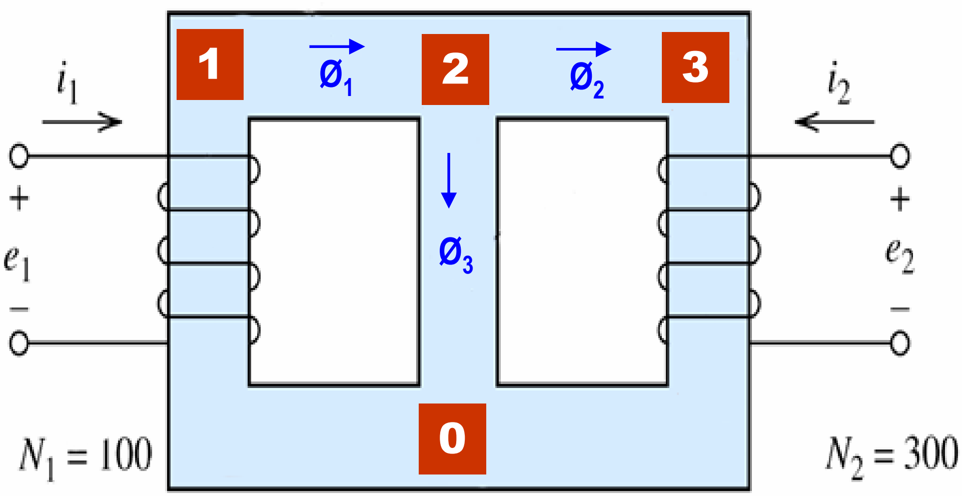
Please note that reluctance is constant in the region below the knee point

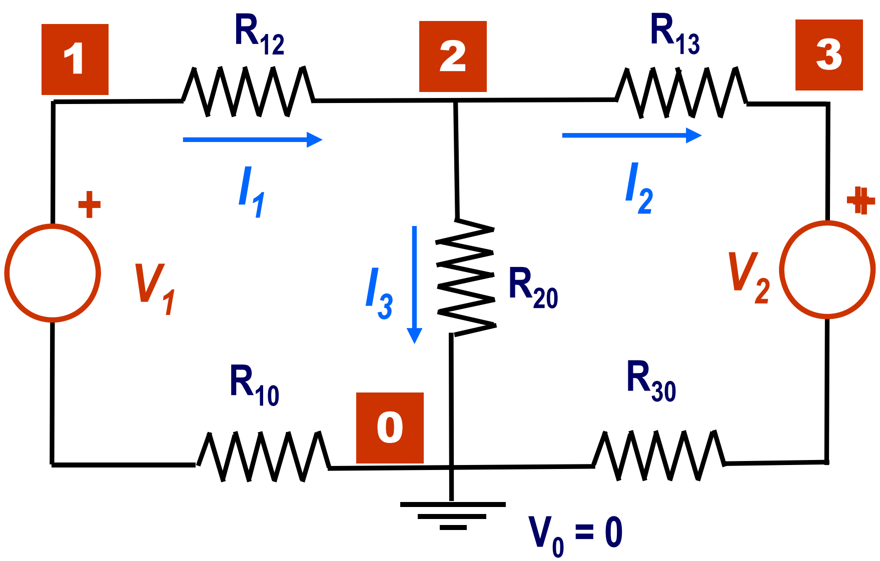
Φ - $\mathcal{F}$ Characteristics



Magnetic Kirchoff's Laws

Kirchoff's First Law	Electrical Analog
Summation of fluxes entering in a junction is equal to that of leaving	$I_1 = I_2 + I_3$ *(Amper)*
$\Phi_1 = \Phi_2 + \Phi_3$ *(Weber)*	

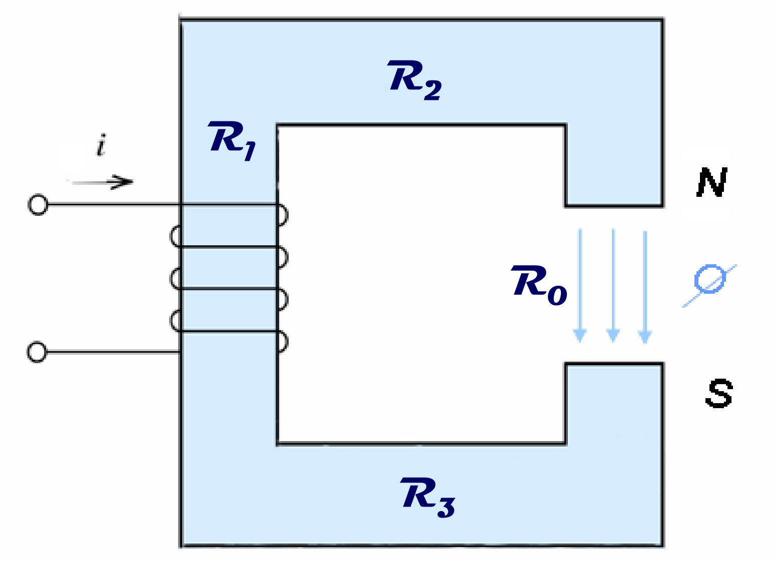


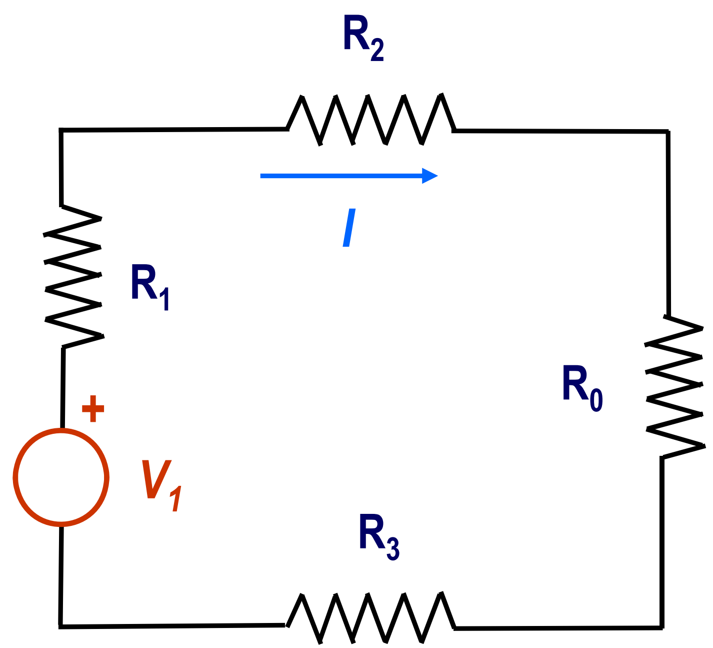


Magnetic Kirchoff's Laws

Kirchoff's Second Law	Electrical Analog
Summation of magneto motive forces in a closed magnetic circuit is zero	$V = R_1 I + R_2 I + R_0 I + R_3 I$

$$\mathcal{F} = NI = \mathcal{R}_1\, \Phi + \mathcal{R}_2\, \Phi + \mathcal{R}_0\, \Phi + \mathcal{R}_3\, \Phi$$





Magnetic Kirchoff's Laws: Application

Kirchoff's Second Law

Reluctance of the parts of the magnetic material

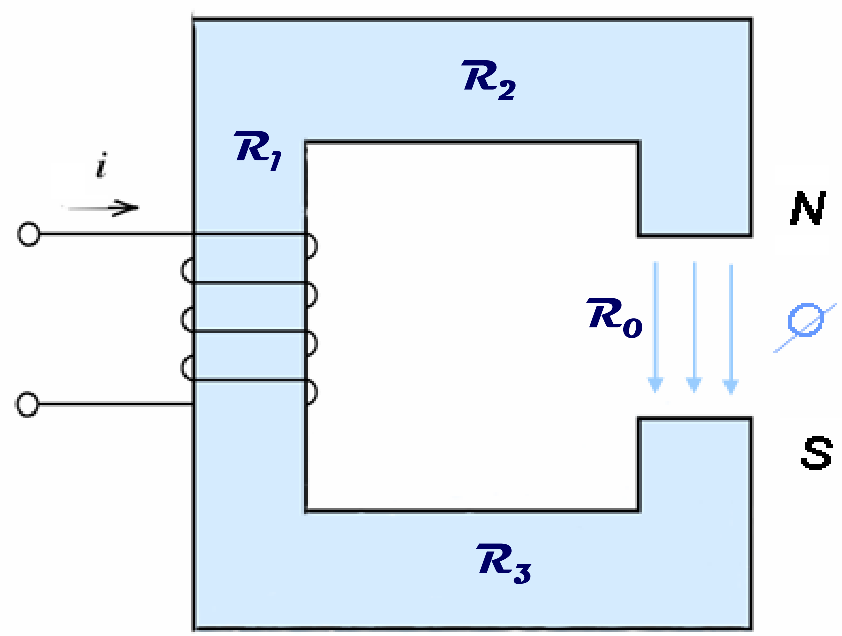
$$\mathcal{R}_1 = \ell_1 / \mu A$$
$$\mathcal{R}_2 = \ell_2 / \mu A$$
$$\mathcal{R}_3 = \ell_3 / \mu A$$

Reluctance of the air gap

$$\mathcal{R}_0 = \ell_0 / \mu_0 A$$
$$\mu_0 = 4 \pi 10^{-7}$$

Total reluctance

$$\mathcal{R}_T = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_0$$



Magnetic Kirchoff's Laws: Application

Kirchoff's Second Law

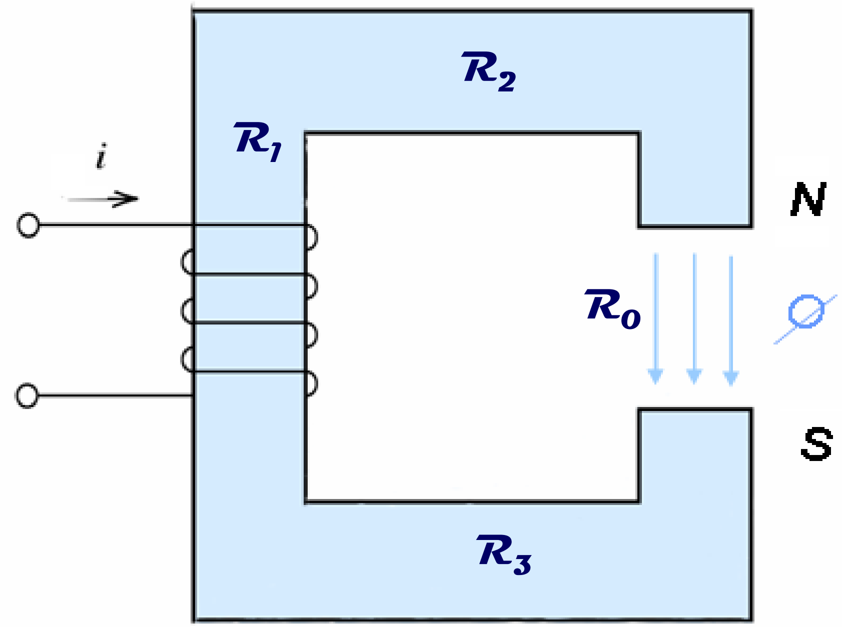
Then flux may be calculated as;

$$\Phi = \mathcal{F} / \mathcal{R}_T$$
$$= NI / (\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_0)$$

or

Then flux may be calculated as;

$$\mathcal{F} = \Phi(\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_0)$$
$$= \Phi\,\mathcal{R}_1 + \Phi\,\mathcal{R}_2 + \Phi\,\mathcal{R}_3 + \Phi\,\mathcal{R}_0$$
$$= \mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_3 + \mathcal{F}_0$$



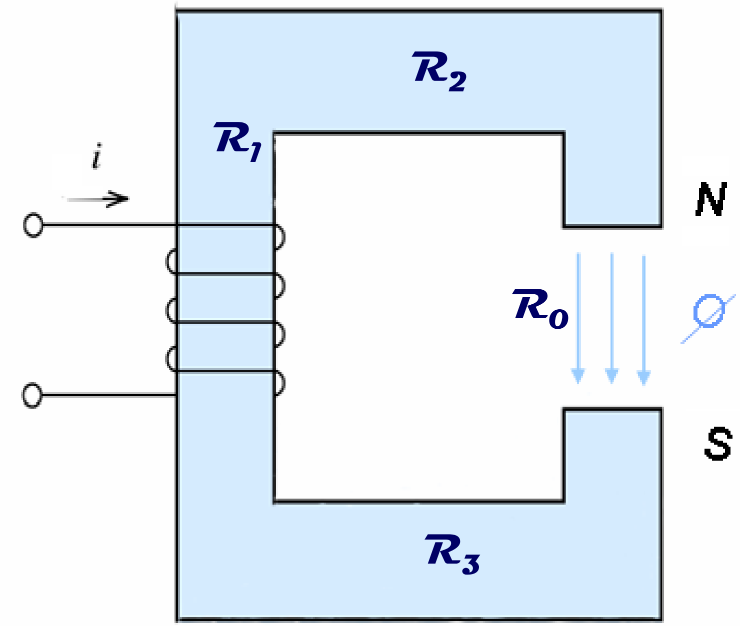
KVL for magnetic circuits

Magnetic Kirchoff's Laws: Application

Kirchoff's Second Law

Then flux density may be written as;

$$B = \Phi / A$$
$$= (\mathcal{F} / (\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_0)) / A$$
$$= (NI / A)/(\ell_1/A\mu + \ell_2/A\mu + \ell_3/A\mu + \ell_0/A\mu_0)$$
$$= NI / (\ell_1/\mu + \ell_2/\mu + \ell_3/\mu + \ell_0/\mu_0)$$



Magnetomotive force may then be written as;

$$\mathcal{F} = NI = \mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_3 + \mathcal{F}_0$$
$$= H_1 \ell_1 + H_2 \ell_2 + H_3 \ell_3 + H_0 \ell_0$$

Ampere's Law: $\oint \mathbf{H} \cdot d\mathbf{l} = \sum Ni$

Magnetic Circuits

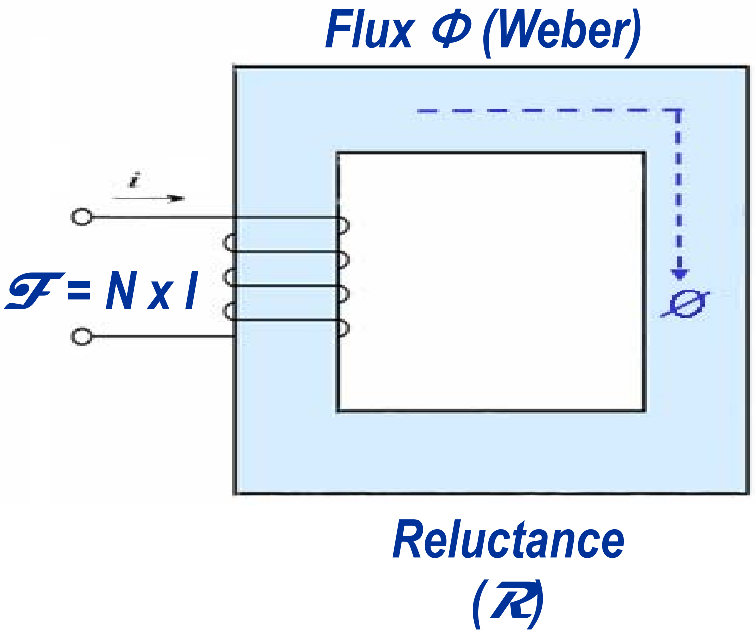
Example 1

Question

The magnetic circuit shown on the RHS is operated at saturated flux values, hence linear techniques can not be applied.
For a flux density of 1.6 Weber / m^2, determine the current *I* and flux *Φ*

Parameters

Cross sectional area = A = 10 cm^2
Mean length of flux path = ℓ_m = 30 cm
Number of turns = N = 500



Magnetic Circuits

Example 1

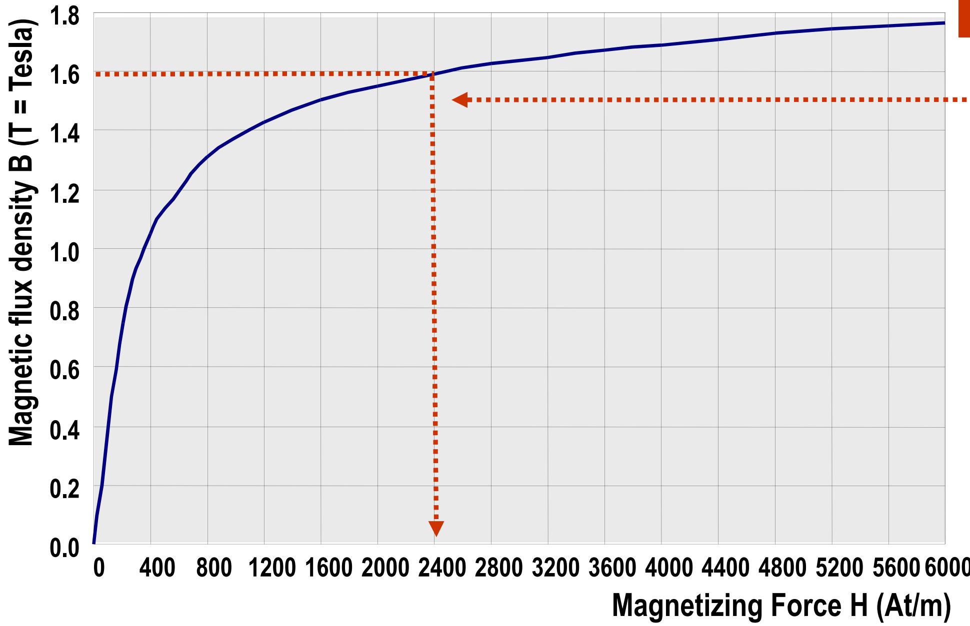
Solution

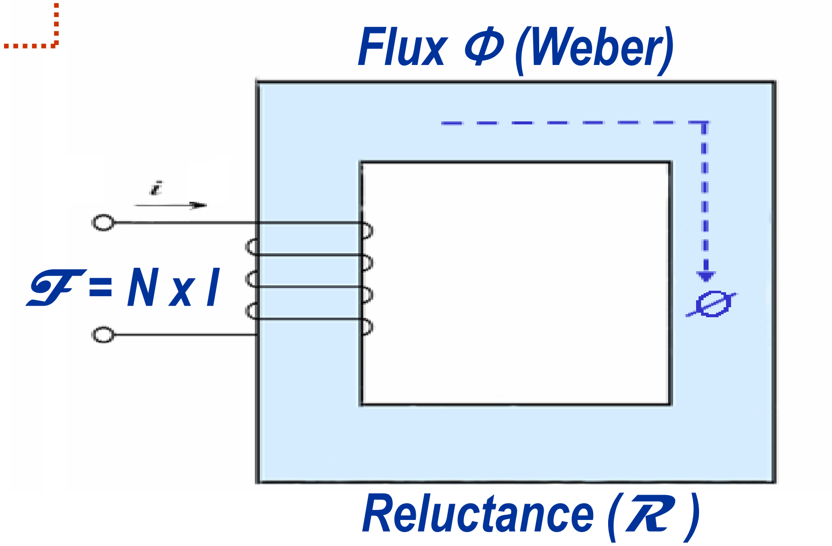
The first thing to do, is to use B-H curve for finding the value of H coreesponding to flux density of 1.6 Weber / m^2

Parameters

Cross sectional area = A = 10 cm^2
Mean length of flux path = ℓ_m = 30 cm
Number of turns = N = 500

B = 1.6 Wb/m^2 $\longrightarrow$ H = 2400 AT / m

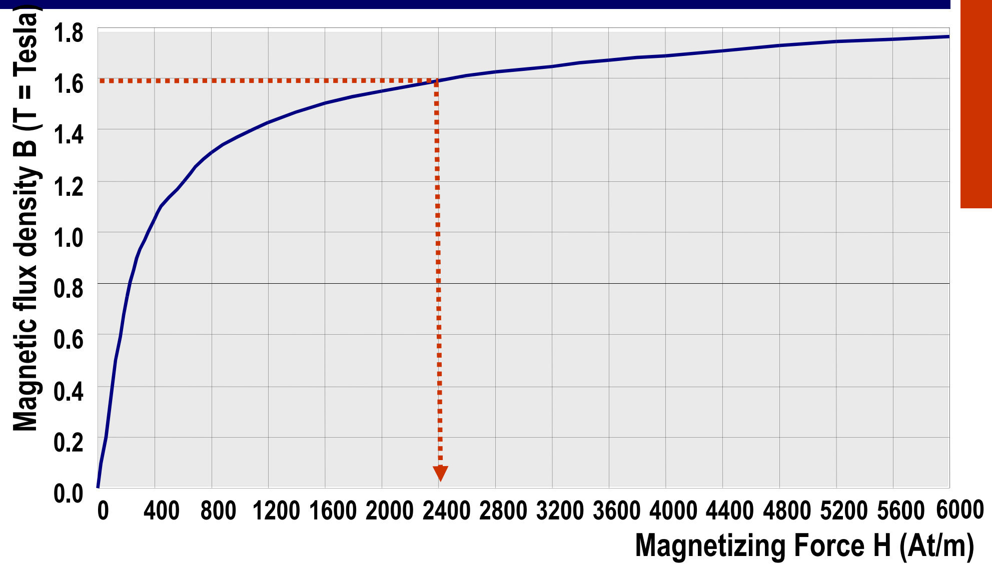




Magnetic Circuits

Example 1

Solution



$B = 1.6\ Wb/m^2 \rightarrow H = 2400\ AT/m$

$\mathcal{F} = NI = H\,\ell_m \rightarrow I = H \times \ell_m / N$

$= 2400 \times 0.3 / 500$

$= 1.44$ Amper

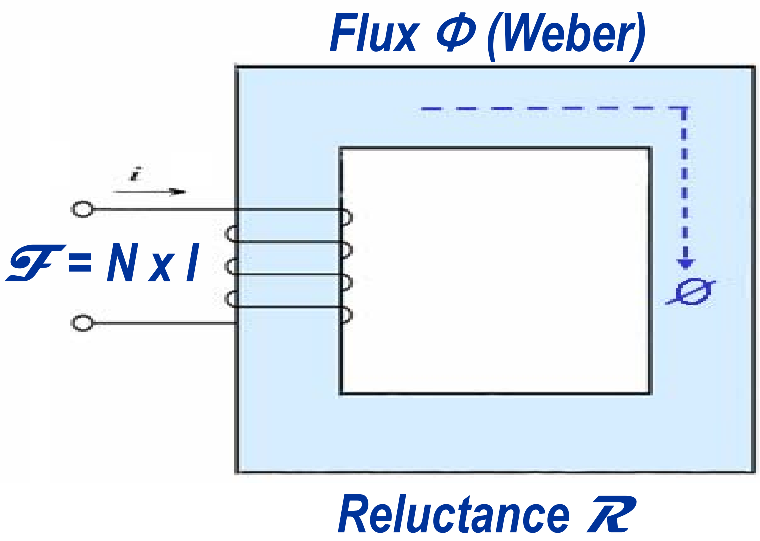
$\Phi = B \times A = 1.6 \times 10 \times 10^{-4} = 0.0016\ Wb$

Parameters

Cross sectional area = A = 10 cm^2

Mean length of flux path = ℓ_m = 30 cm

Number of turns = N = 500



Magnetic Circuits

Example 2

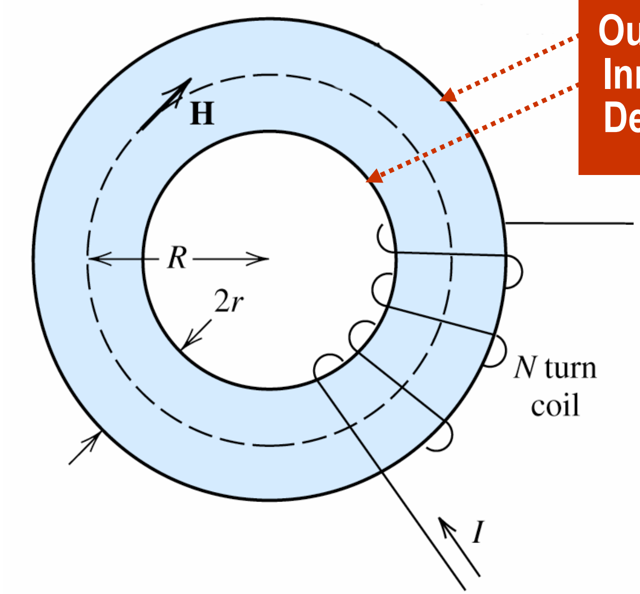
Question

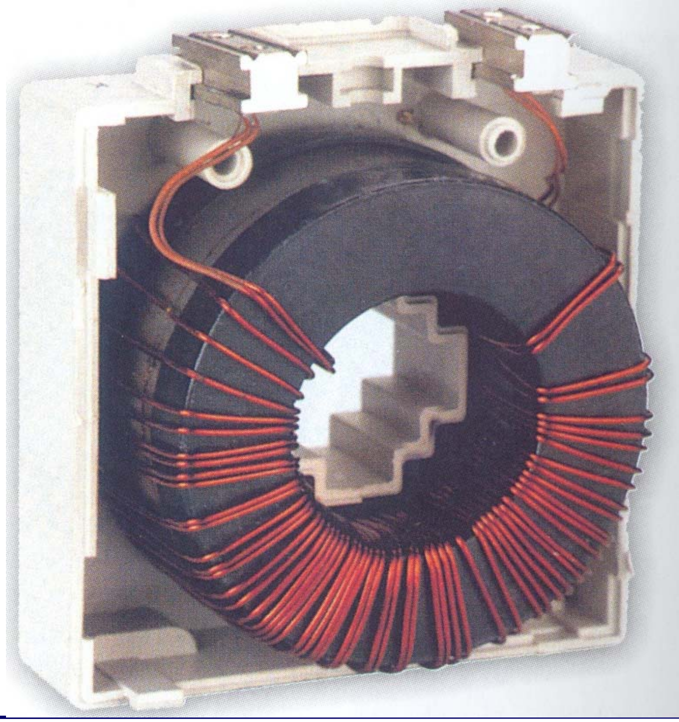
Determine the flux and mmf required to produce the flux in the toroid shown on the RHS wound around a certain ferromegnetic material called "Ferrite"

Flux density = B = 0.15 Wb / m^2
$\mu = 1000 \times \mu_0$ (B-H curve is linear)

The ratio μ / μ_0 is called "Relative Permeability"

Outer diamater = 0.10 m
Inner diameter = 0.08 m
Dept = 0.02 m





Example 2

Solution

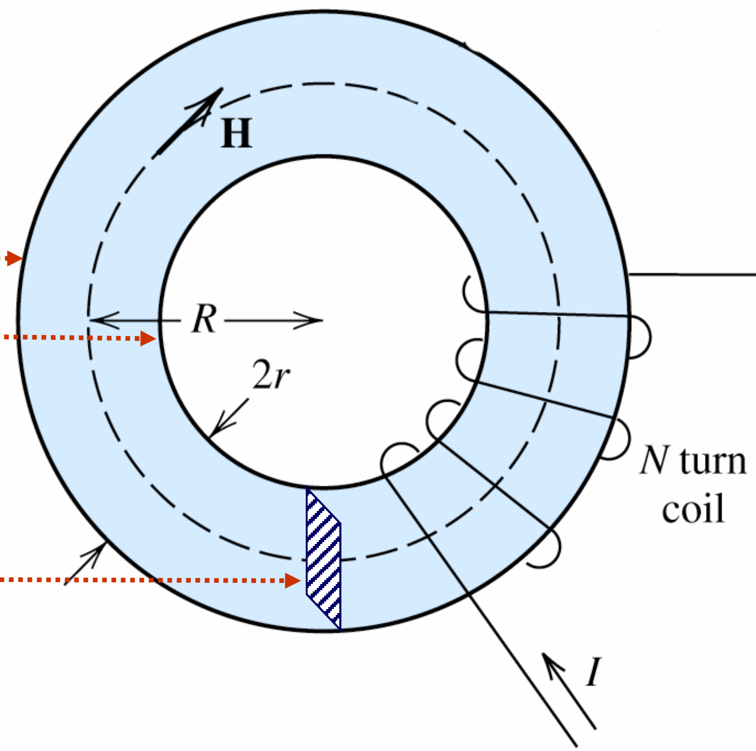
Thickness of the core = (0.10 – 0.08) / 2 = 0.01 m
Diameter_{mean} = (0.10 + 0.08) / 2 = 0.09 m

Cross sectional area = 0.02 x 0.01 = 0.0002 m^2
Then, flux = Φ = B x A = 0.15 x 0.0002 = 0.00003 Wb

Flux density = B = 0.15 Wb / m^2
μ = 1000 x μ_0
(B-H curve is linear)

Outer diamater = 0.10 m
Inner diameter = 0.08 m
Dept = 0.02 m

Cross sectional area A



Magnetic Circuits

Example 2

Solution

Let us now calculate the mmf necessary for producing this flux

Flux density = B = 0.15 Wb / m^2
$\mu = 1000 \times \mu_0$
(B-H curve is linear)

Reluctance of the toroid $= \mathcal{R} = \ell_{mean} / \mu A$

Where ℓ_{mean} *is the mean length defined as;*

ℓ_{mean} = diameter $_{mean} \times \pi$

$= 0.09 \times \pi = 0.2827$ m

Hence the reluctance becomes;

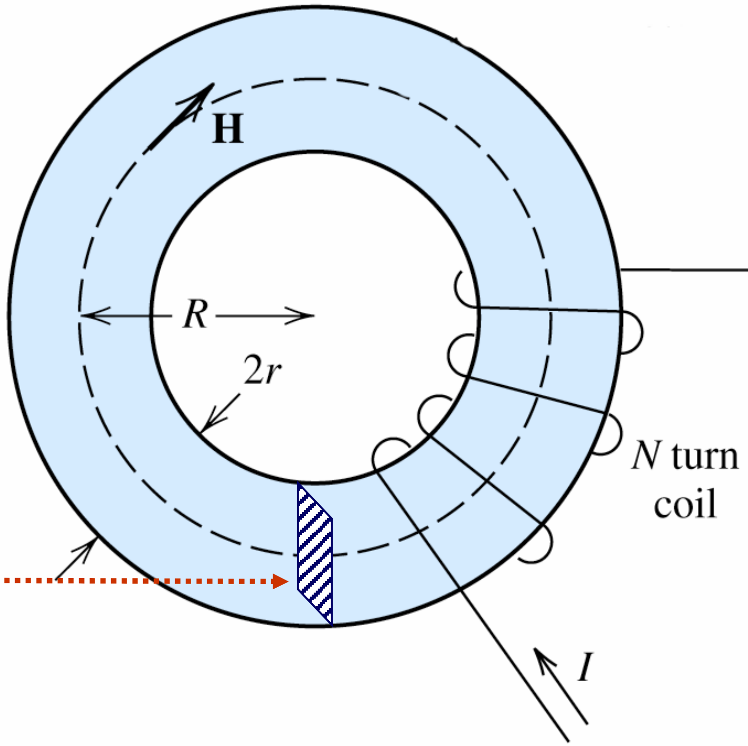
$\mathcal{R} = \ell_{mean} / \mu A$

$= 0.2827 / (1000 \times 4 \pi \times 10^{-7})$

$= 224.97$ AT / Wb

Thus, mmf becomes;

$\mathcal{F} = \mathcal{R}\, \Phi = 224.97 \times 0.15 = 33.75$ AT



Magnetic Circuits

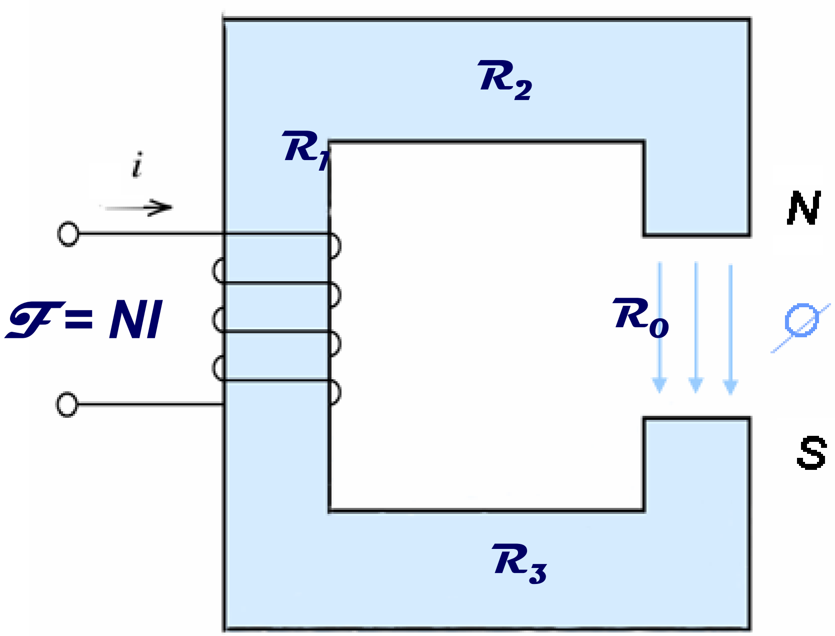
Load Line Technique

The need for the Load Line Technique

When the material is operated within the saturated region of the B-H characteristics, solution of the circuit when the mmf is given, but the flux is unknown is rather difficult, due to the fact that μ of the material is a funciton of flux Φ, but flux is unknown, i.e. the equation;

$$\Phi = \mathcal{F} / (\mathcal{R}_{total} + \mathcal{R}_0) = \mathcal{F} / (\ell_{total} / (\mu(\Phi) \times A)) + \mathcal{R}_0)$$

Implicit nonlinar equation



In other words, both sides of the above equation involve the unknown variable Φ

Magnetic Circuits

Example 4

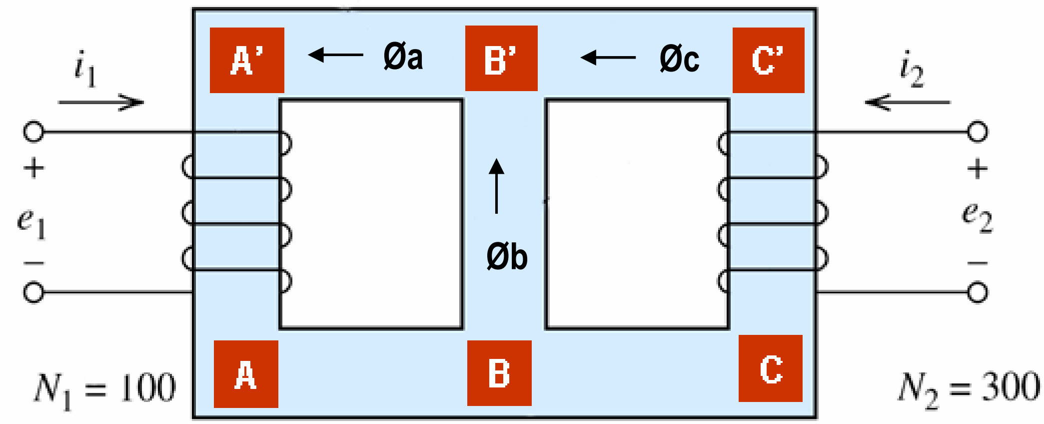
Question

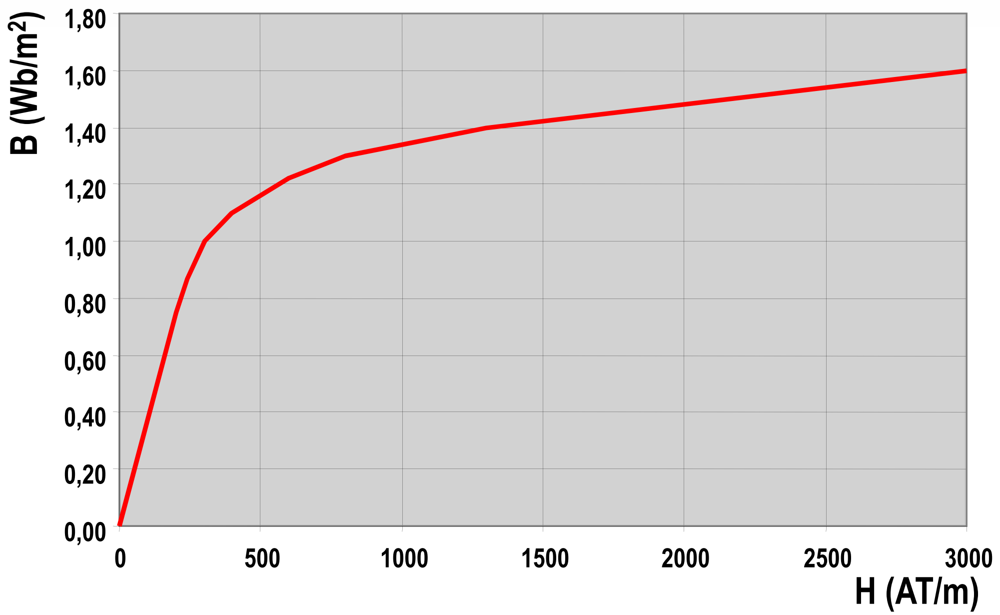
B-H curve of the magnetic circuit shown on the RHS is given below. Fluxes in the branches are given as;

$Ø_a$ = 0.003 Weber,
$Ø_b$ = 0.003 Weber,
$Ø_c$ = 0.003 Weber

find the direction and magnitudes of the currents

$\ell_{BB'}$ = 0.1 m
$\ell_{BAA'B'}$ = $\ell_{BCC'B'}$ = 0.4 m
$A_{BB'}$ = 5 cm^2
$A_{BAA'B'}$ = $A_{BCC'B'}$ = 20 cm^2





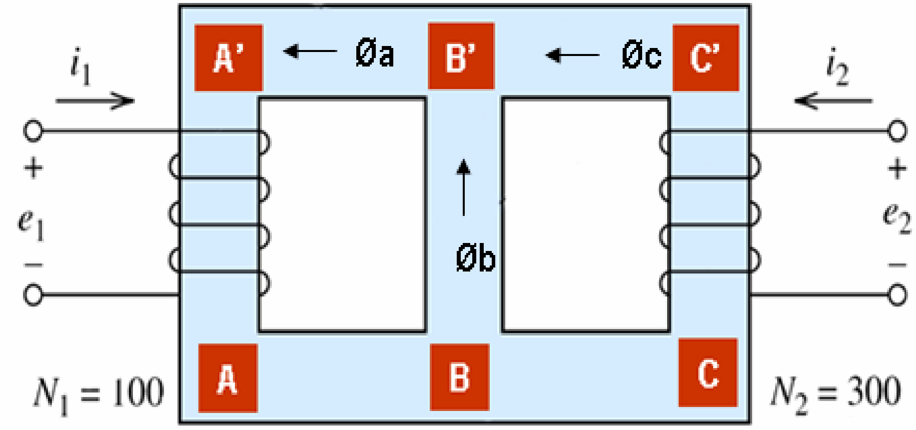
Solution

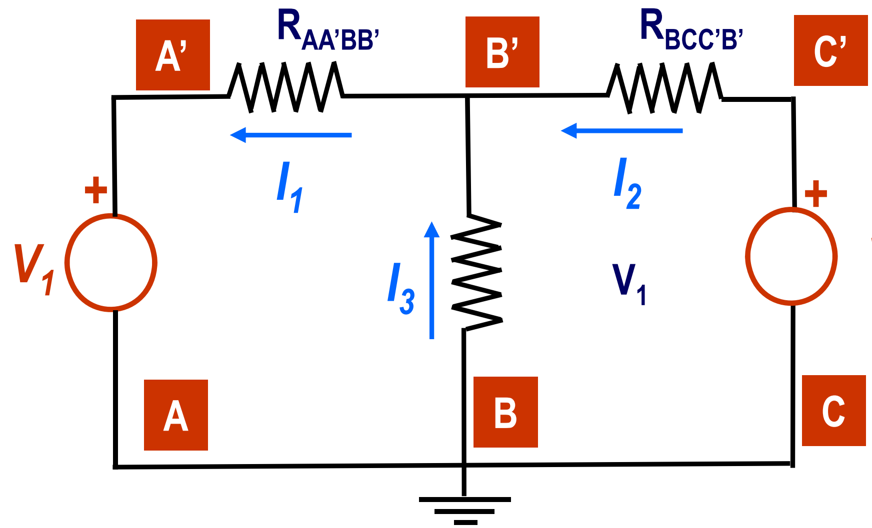
Solution

Writing down magnetic KVL for the loops of the magnetic circuit

$$\mathcal{F}_1 = N_1 I_1 = H_{BB'}\,\ell_{BB'} + H_{BAA'B'}\,\ell_{BAA'B'} \quad (1)$$

$$\mathcal{F}_2 = N_2 I_2 = H_{BCC'B'}\,\ell_{BCC'B'} + H_{BB'}\,\ell_{BB'} \quad (2)$$





Solution

Solution

Now, flux densities

$B_{BB'} = Ø_b / A_{BB'} = 0.0008 / (5 \times 10^{-4})$
$= 1.6 \ Wb/m^2$

$B_{BAA'B'} = Ø_a / A_{BAA'B'} = 0.0030 / (20 \times 10^{-4})$
$= 1.5 \ Wb/m^2$

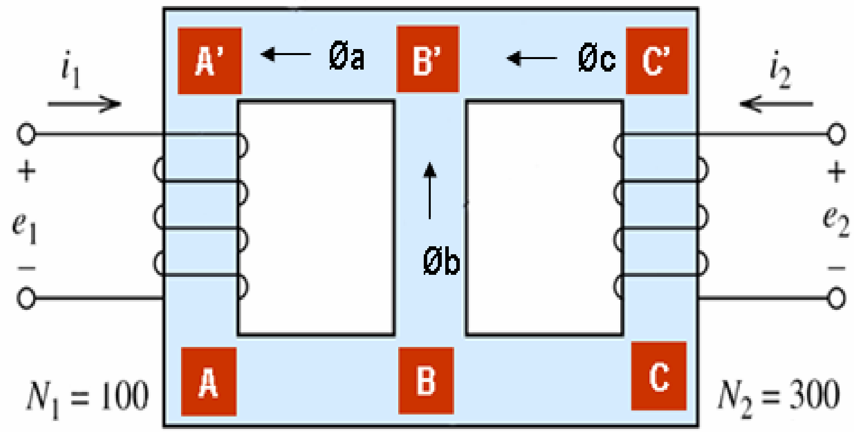
$B_{BCC'B'} = Ø_c / A_{BCC'B'} = 0.0022 /(20 \times 10^{-4})$
$= 1.1 \ Wb/m^2$

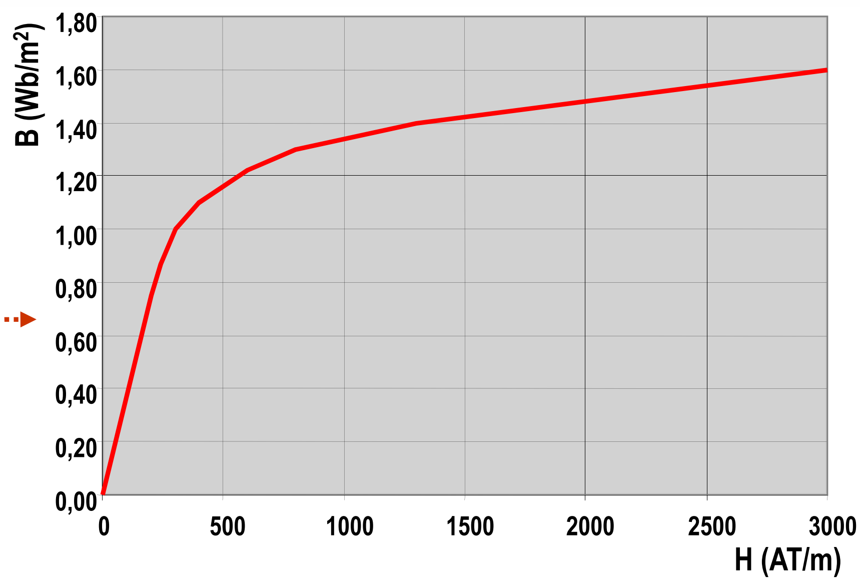
Now, find magnetizing forces by using the given B-H curve

$B_{BB'} = 1.6 \ Wb/m^2 \Rightarrow H_{BB'} = 3000$ AT/m

$B_{BAA'B'} = 1.5 \ Wb/m^2 \Rightarrow H_{BAA'B'} = 2000$ AT/m

$B_{BCC'B'} = 1.1 \ Wb/m^2 \Rightarrow H_{BCC'B'} = 400$ AT/m





Magnetic Circuits

Solution

Solution

Now, substitute the above magnetizing forces into equatins (1) and (2)

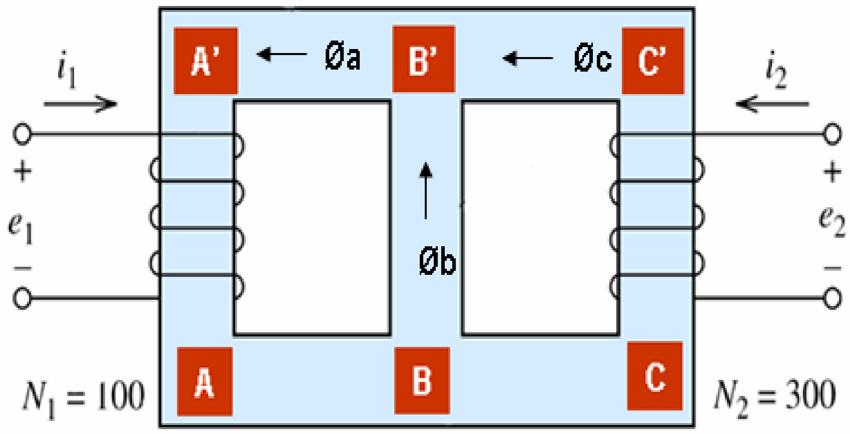
$100\ I_1 = 3000 \times 0.01 + 2000 \times 0.4 = 1100$

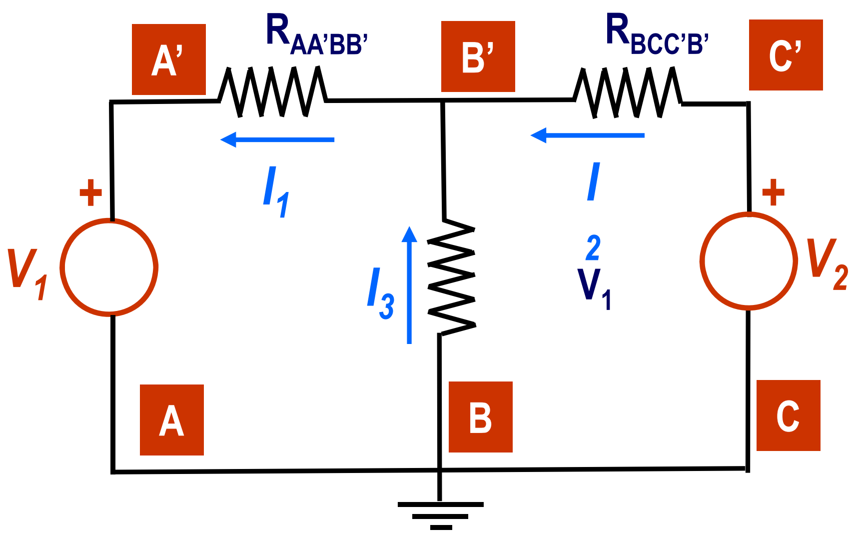
Thus, $I_1 = 1100 / 100 =$ 11.0 Ampers

$300\ I_2 = 400 \times 0.4 - 3000 \times 0.1 = -140$

Thus $I_2 = -140.0 / 300 =$ - 0.467 Ampers

Please note that I_1 is in the same direcition as shown in the figure, but I_2 is in the opposite direction





Graphical Solution (Load Line Technique)

Description

Writing Amper's equation for the magnetic circuit

$$\mathcal{F} = H_c\,\ell_c + H_0\,\ell_0 \qquad (1)$$

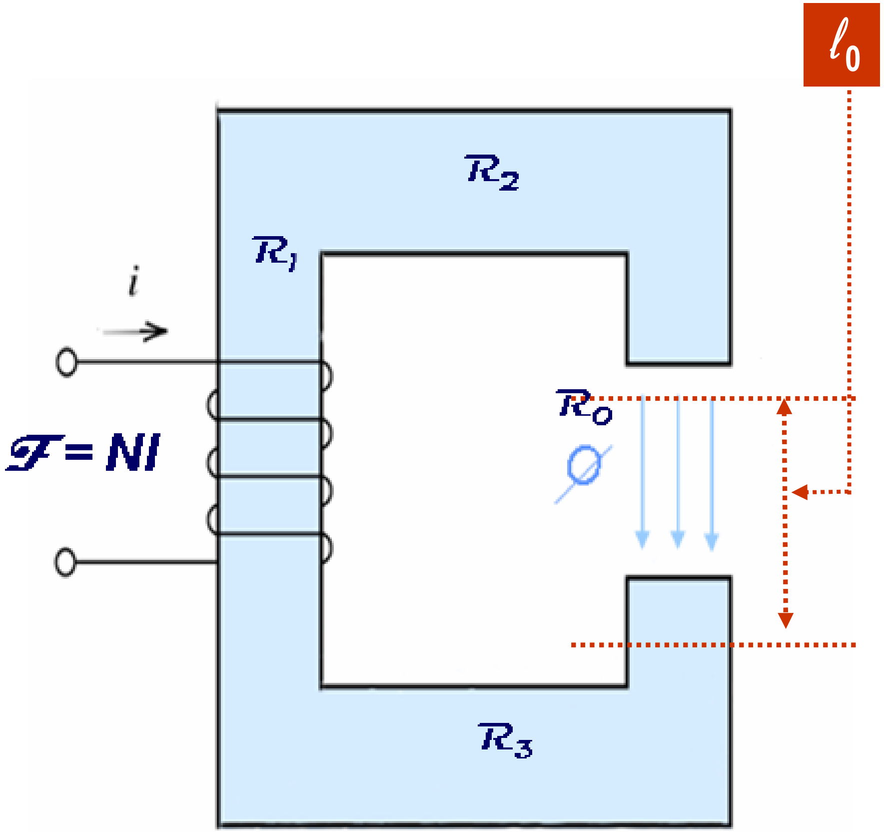
where, H_c and H_0 are the magnetizing forces in the material (core) and air gap,

ℓ_c and ℓ_0 are the lengths of the core and air gap, respectively

Now, noting that;

$$H_0 = B_0 / \mu_0 = B_c / \mu_0$$

And substituting this into *(1)*



Magnetic Circuits

Graphical Solution (Load Line Technique)

Derivation of the Load Line Equation

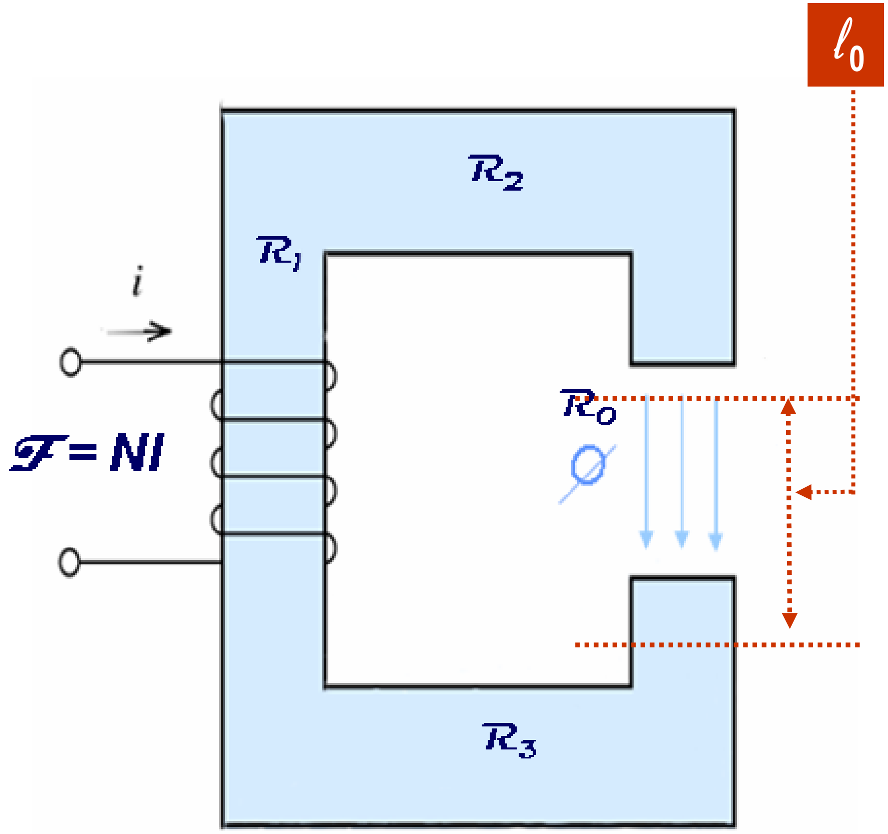
$$\mathcal{F} = NI = H_C\, \ell_c + (B_c / \mu_0)\, \ell_0 \qquad (2)$$

or writing the above equation in terms of H_c

$$H_C = -\ \ell_0 / (\mu_0\ \ell_c)\ \ B_c + NI / \ell_c \qquad (3)$$

Load Line Equation:
$y = -\ a x + b$

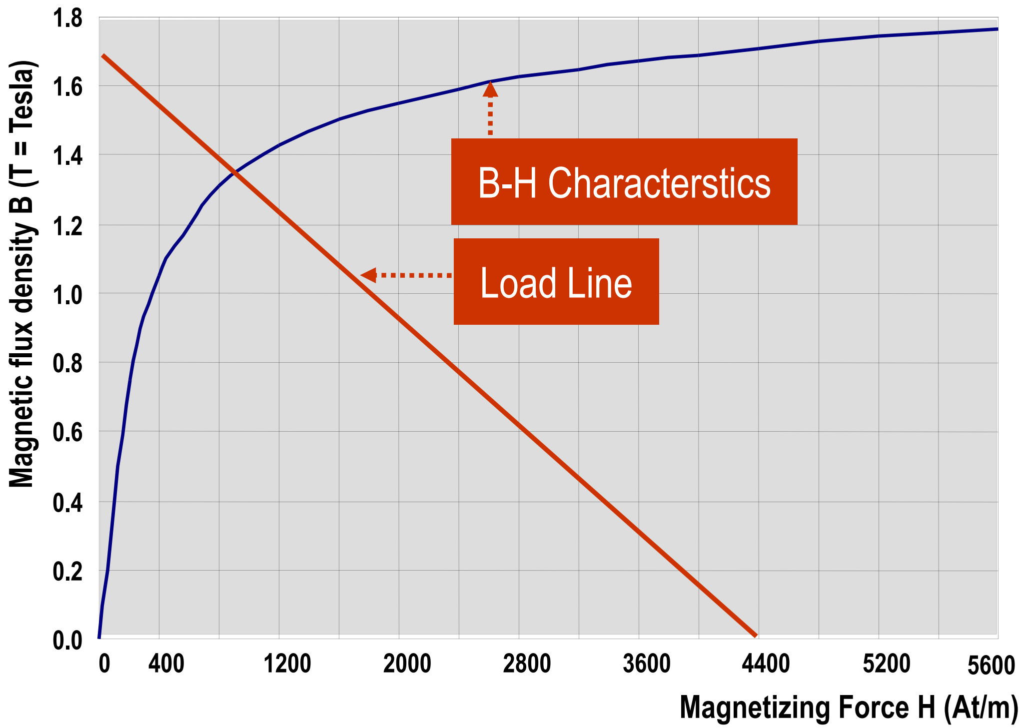
The Load Line is then drawn on the B-H characteristics and the intersection point of the two curves is found

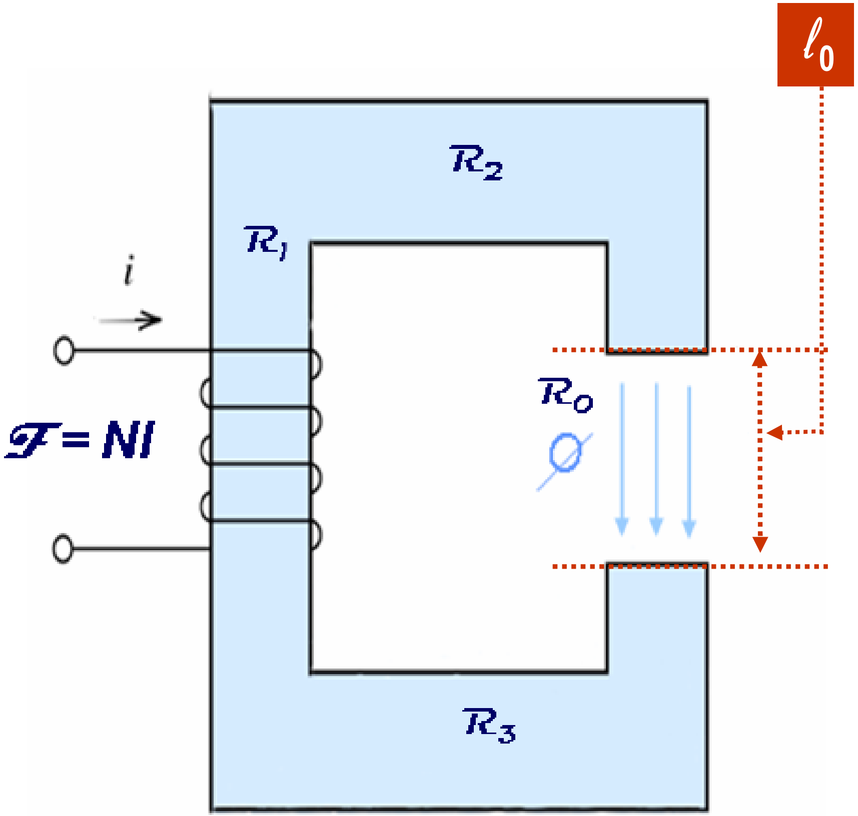


Graphical Solution (Load Line Technique)

Graphical Solution

Solve the magnetic circuit shown on the RHS with the B-H characteristics

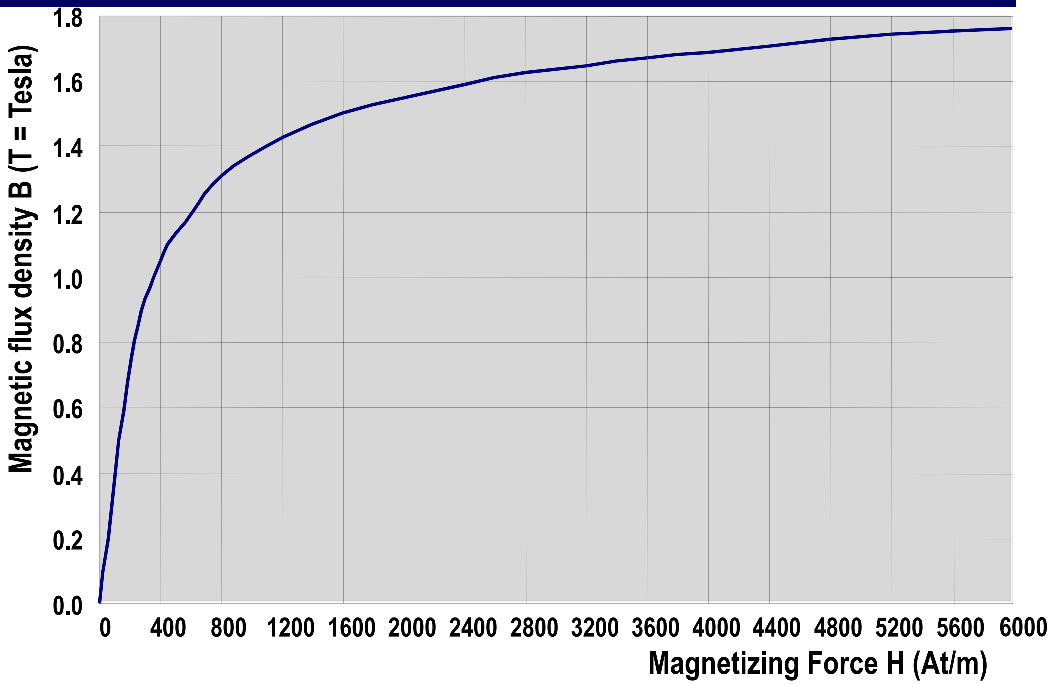




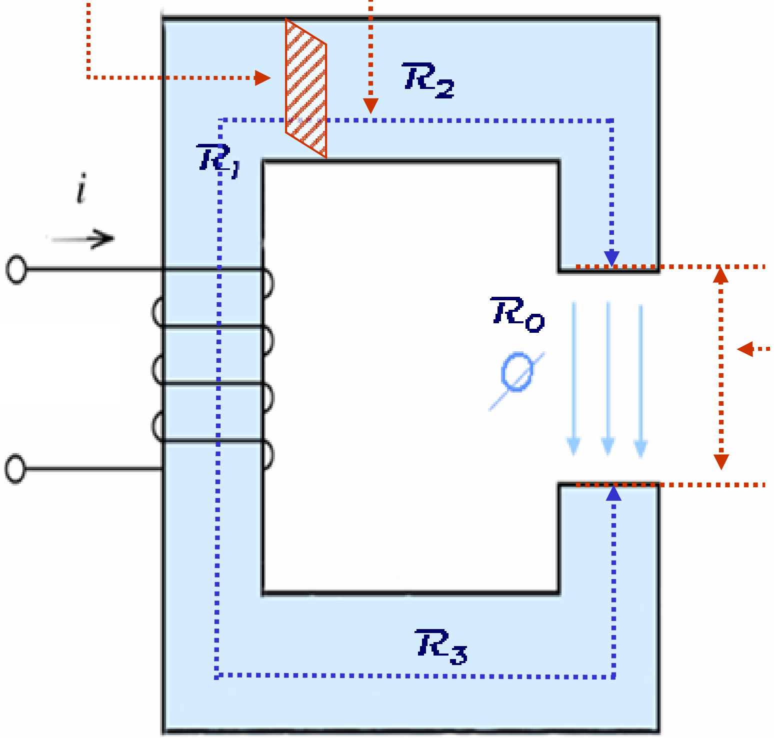
Example 3

Question

Solve the magnetic circuit shown on the RHS with the B-H characteristics given below by using the Load Line Technique



$A_c = 8 \times 10^{-3}$ m | $\ell_c = 0.55$ m | $\ell_0 = 10^{-3}$ m



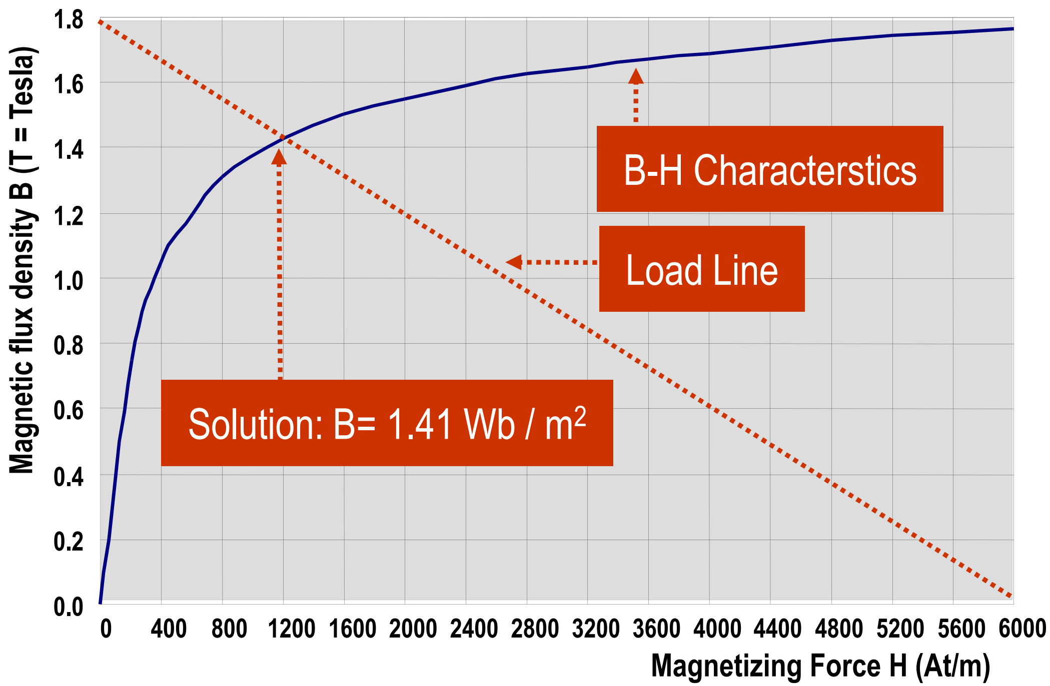
$\mathcal{F} = NI = 3300$ AT

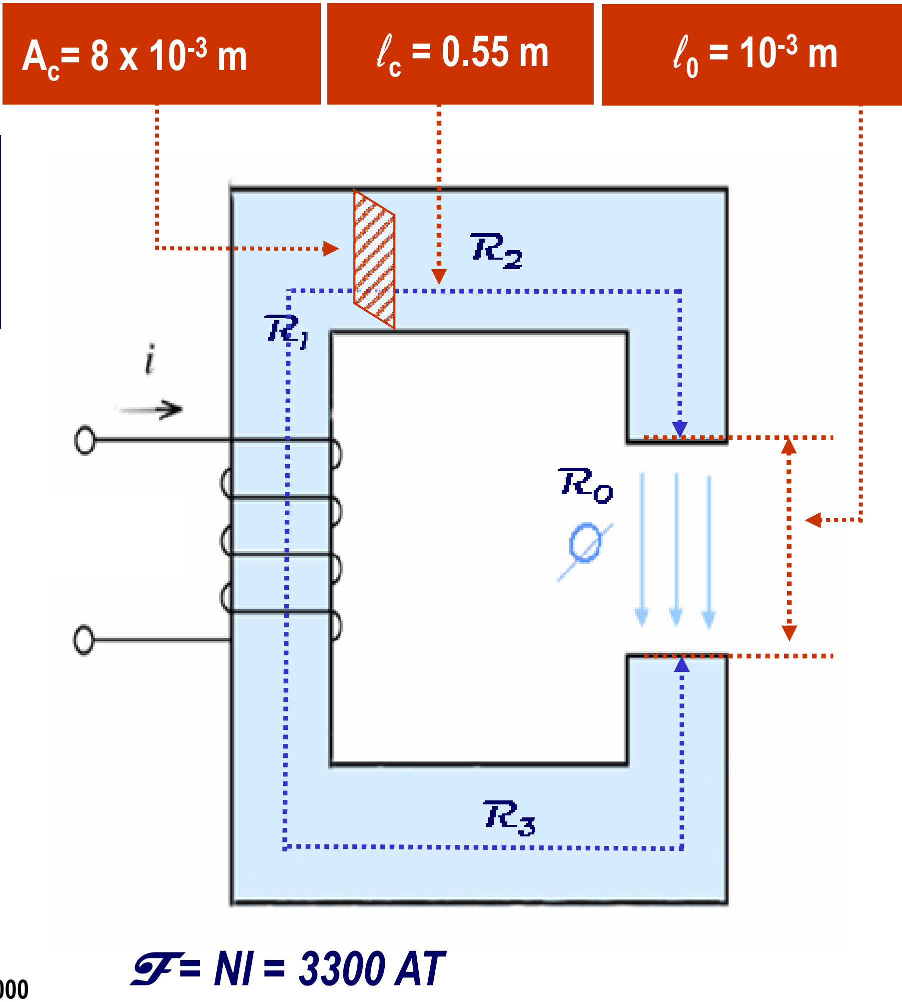
Magnetic Circuits

Example 3

The Load Line Equation

$$H_C = - \ell_0 / (\mu_0\ \ell_c)\ B_c + NI / \ell_c$$
$$= - 3327\, B_c + 6000$$





Energy in Magnetic Circuits

Sinusoidally Excited Magnetic Circuits

Consider the sinusoidally excited magnetic circuit shown on the RHS

$I(t) = -\hat{I}\cos wt$

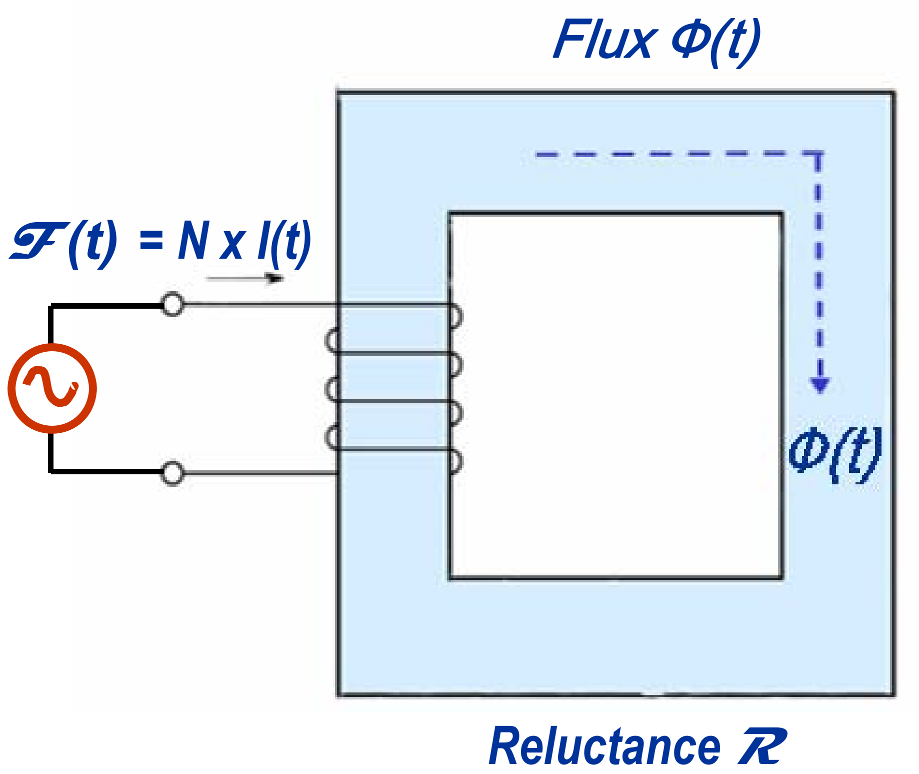
$\mathcal{F}(t) = -N\hat{I}\cos wt$

$$\Phi(t) = \mathcal{F}(t) / \mathcal{R}_{total}$$

$$= -N\hat{I}\cos wt / \mathcal{R}_{total}$$

$$= -\hat{\Phi}\cos wt$$

where $\hat{\Phi} = NI / \mathcal{R}_{total}$



Energy in Magnetic Circuits

Sinusoidally Excited Magnetic Circuits

$$I(t) = -\hat{I}\cos wt$$

$$\mathcal{F}(t) = -N\hat{I}\cos wt$$

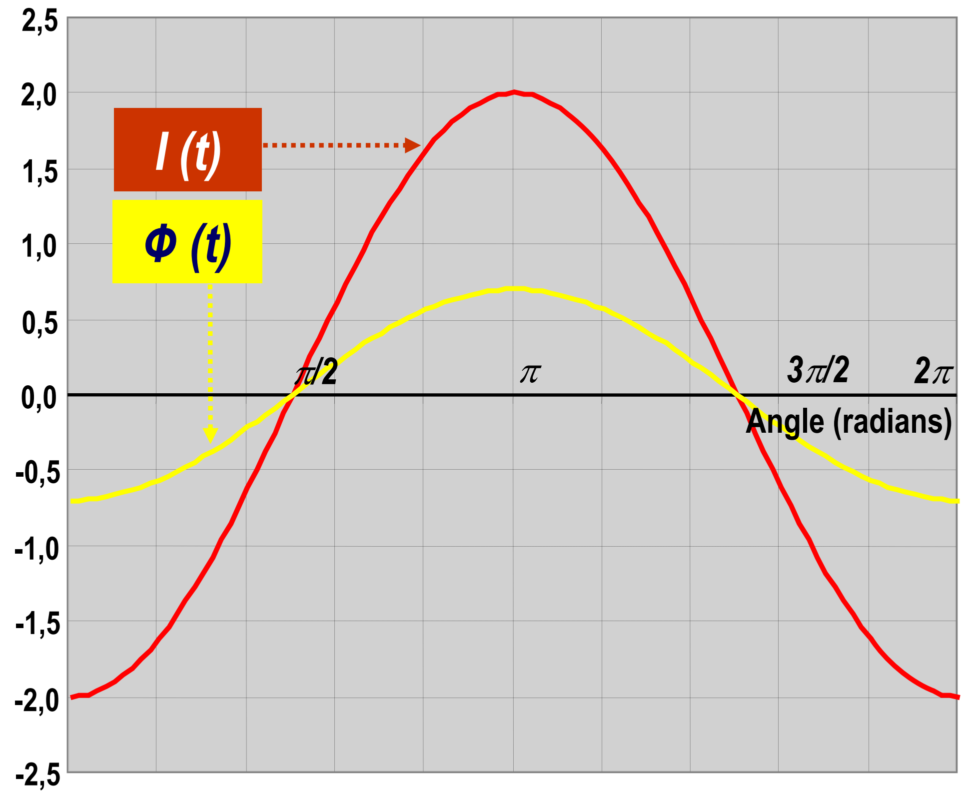
$$\Phi(t) = \mathcal{F}(t) / \mathcal{R}_{total}$$

$$= -N\hat{I}\cos wt / \mathcal{R}_{total}$$

$$= -\hat{\Phi}\cos wt$$

where, $\hat{\Phi} = NI / \mathcal{R}_{total}$

Waveforms



Voltage Induced in the Coil

Sinusoidally Excited Magnetic Circuits

By using Lenz's law

$$e(t) = N\, d/dt\, \Phi(t)$$

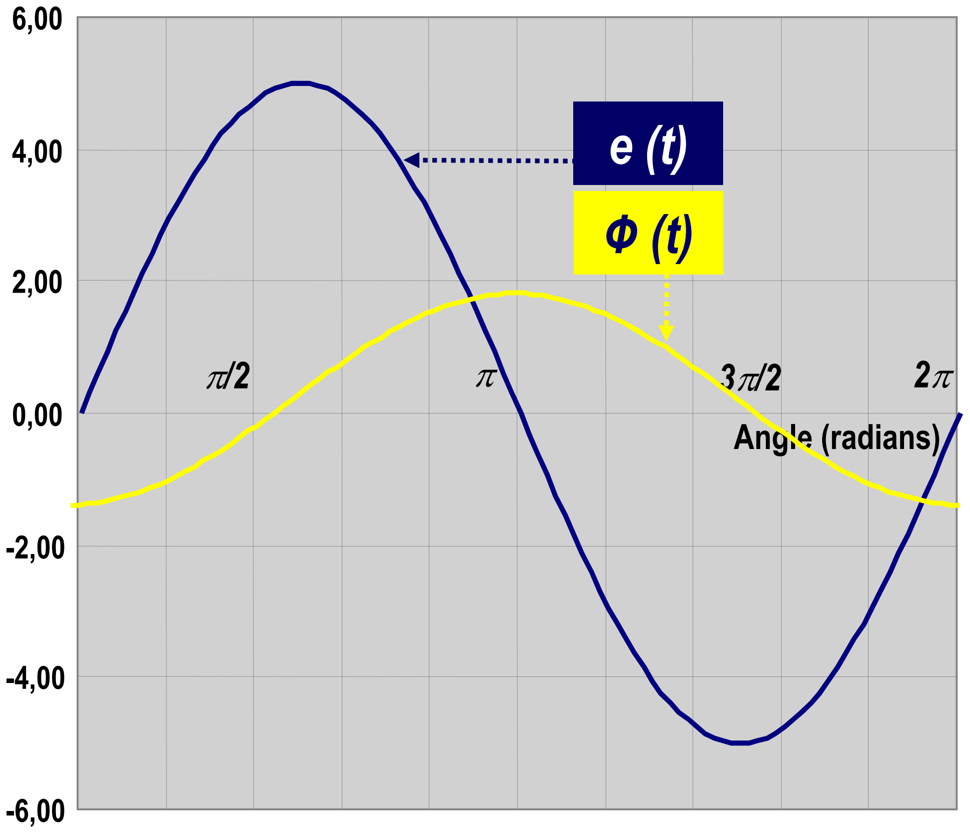
$$= -N\, d/dt\, \hat{\Phi} \cos wt$$

$$= N \hat{\Phi}\, w \sin wt$$

$$= \hat{e} \sin wt$$

where $\hat{e} = N \hat{\Phi}\, w$

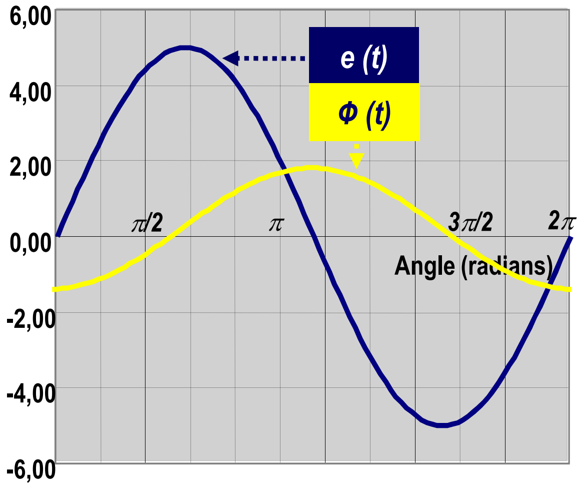
Waveforms

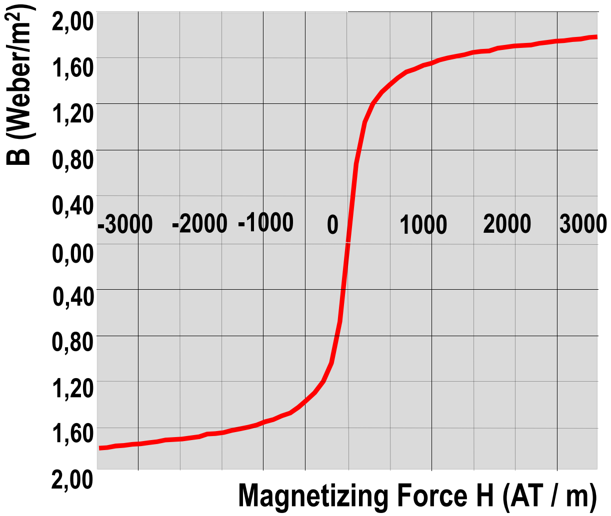


Voltage Induced in the Coil

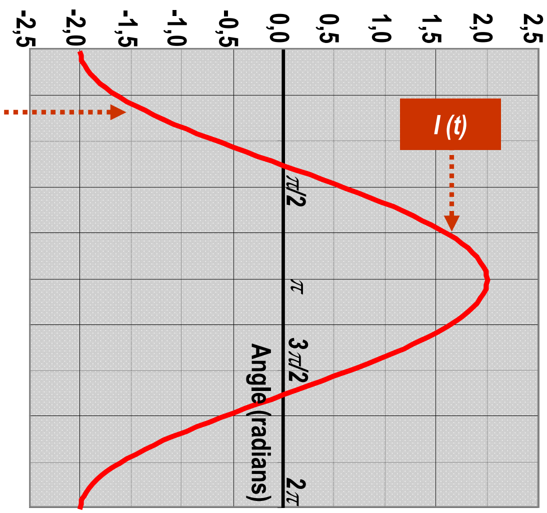
Sinusoidally Excited Magnetic Circuits

If we plot $\Phi(t)$ and $e(t)$ on the same scale





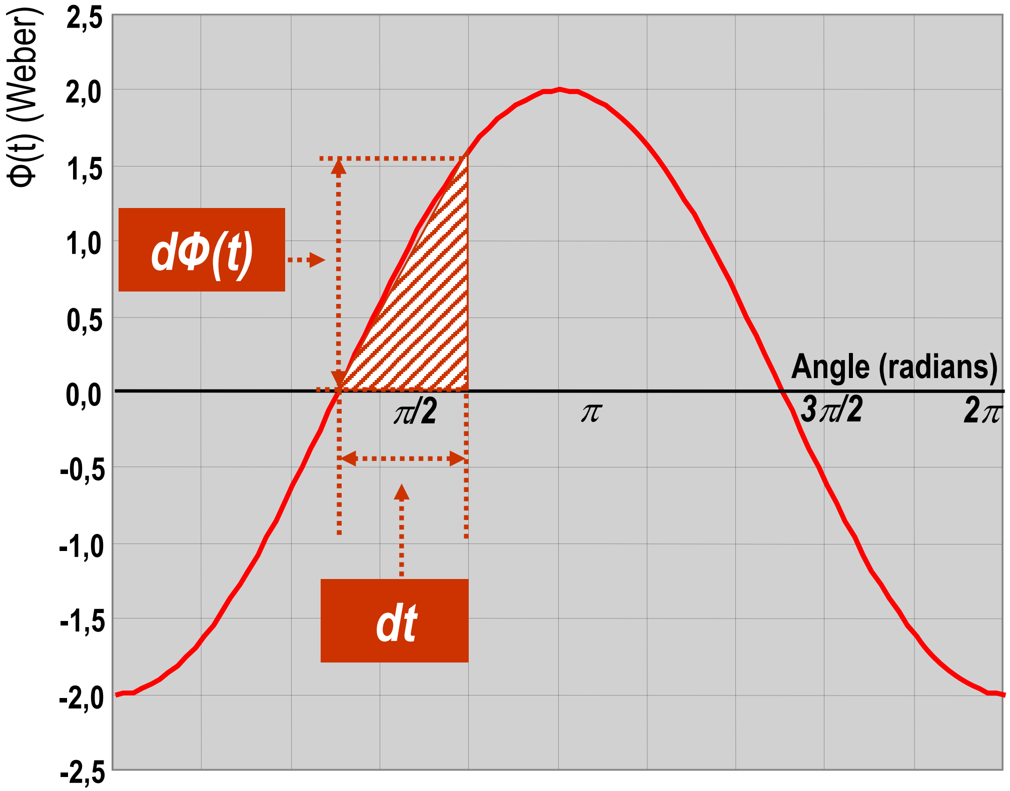
Please note that the shape of this waveform is not actually of pure sinusoidal shape, but a distorded form of sinusoidal waveform, due to nonlinearity of the B-H curve



Energy Stored in Magnetic Circuits

Electrical Energy Stored in a Magnetic Circuit

Time duration *dt* required for a change *dΦ* in flux may be found on the *Φ – t* curve

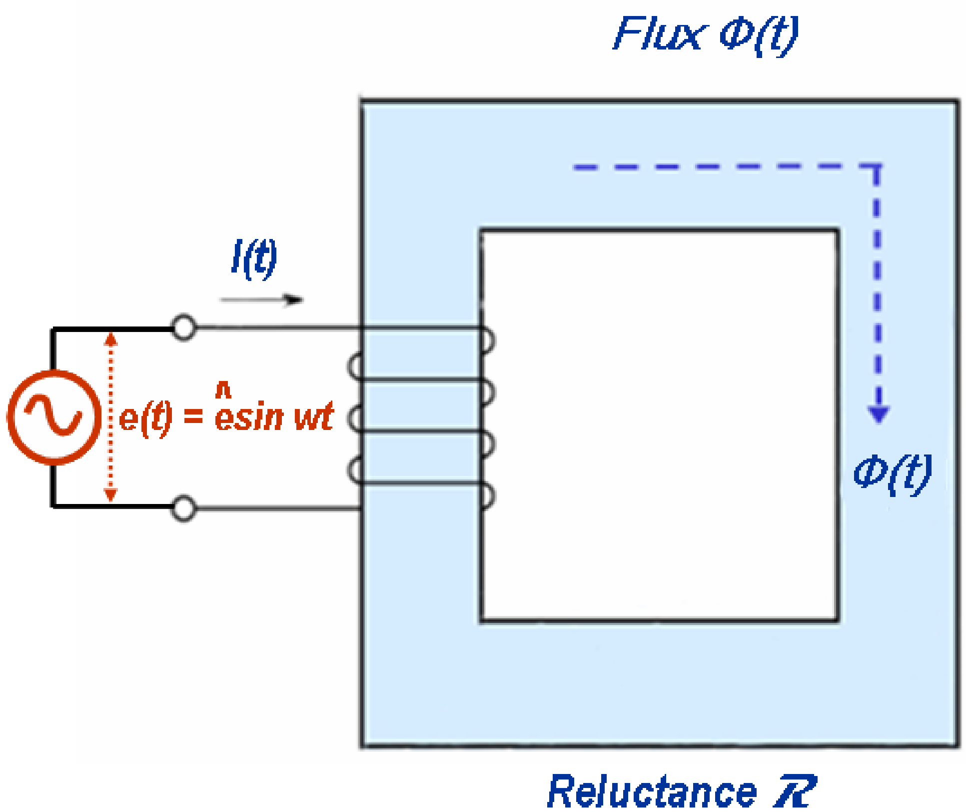


Energy Stored in Magnetic Circuits

Electrical Energy Stored in a Magnetic Circuit

Let us now calculate the instantaneous electrical energy stored in a magnetic circuit within a time duration dt

$$dW_{elect} = e(t)\ I(t)\ dt$$
$$= N\ d\Phi(t)/dt\ \ I(t)\ dt$$
$$= (NI)\ d\Phi(t)\ /\ \cancel{dt}\ \times\ \cancel{dt}$$
$$= \mathcal{F}\ d\Phi$$



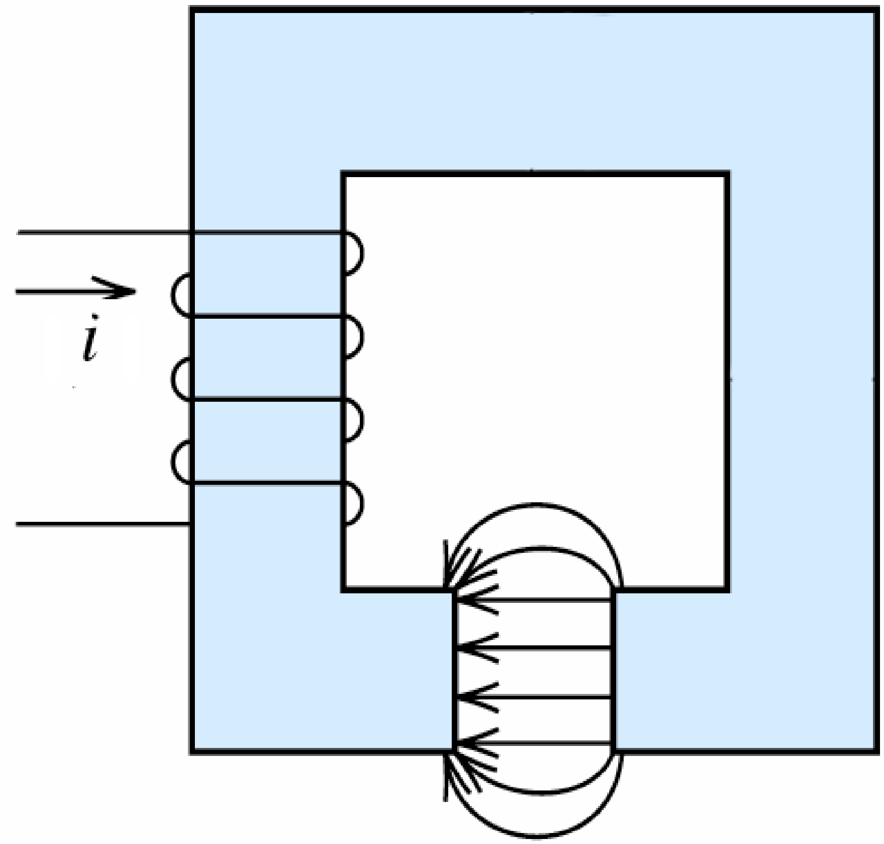
Fringing Effect

Description

Fringing effect is the deviation of the flux trajectory to outside in the air gap

The effects of fringing;
(a) increasing the cross sectional area of the air gap,
(b) creating nonlinarity in the flux density in the air gap

Usual practice for handling the fringing effect is to increase the cross sectional area of the air gap in calculations by a factor, such as 20 %



Magnetic Circuits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Any Questions Please ?